McMaster University

Hyperbolic Conduction

Going Beyond Euler

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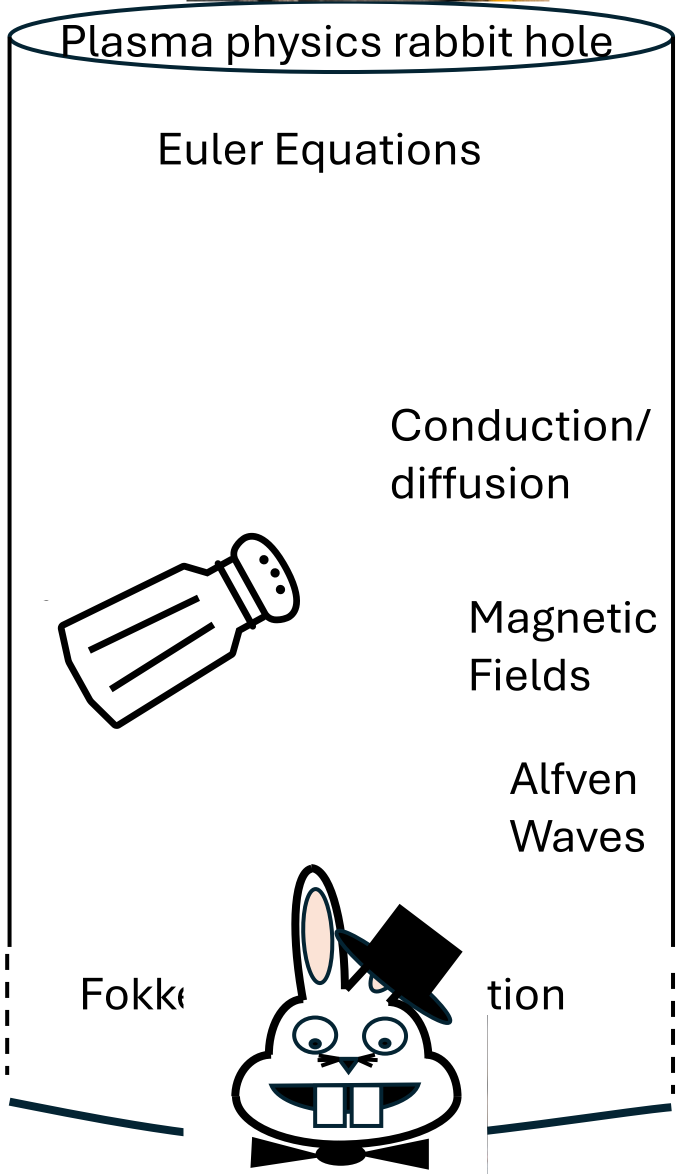
Introduction

- **Fluid regime:** Euler equations standard for astrophysical gas simulations.
  - Continuum approximation
    no viscosity, no conduction
  - Perfect fluid

- Real astrophysical plasmas (low density/high temperature) progressively break these assumptions
  e.g. supernova explosions

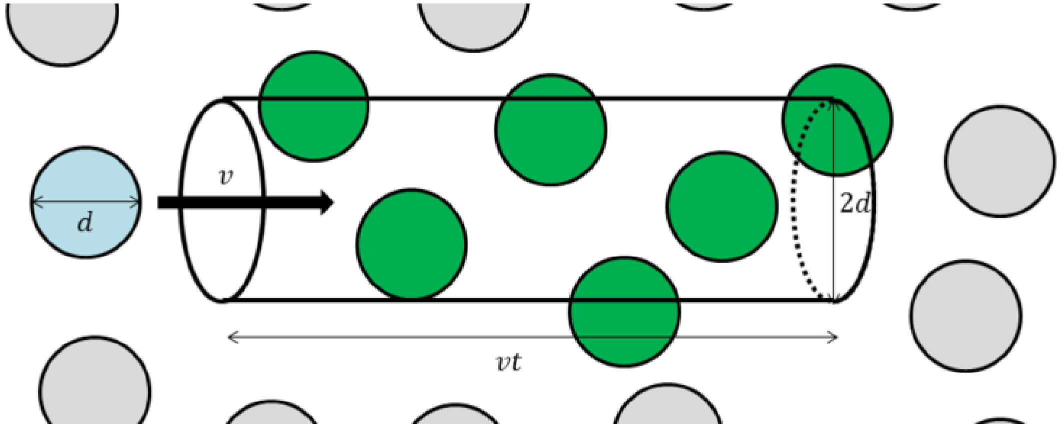
- **Kinetic regime:** Ultimate approach:
  full collisional Boltzmann equation
  Model individual particles with collisions (6D)

- Additional components: magnetic fields, relativistic particles (cosmic rays).



What makes something a Fluid?
Mean free path (MFP, λ)

- Fluid regime: **Continuum approximation** requires frequent collisions.
- If collisions happen ***well*** below relevant physical scale, **L** : fluid regime



Solo5010 via Wikimedia commons

- Neutral gas: MFP inversely proportional to density.

$$\lambda \sim 1 / n$$

- Ionized: MFP proportional to temperature squared as well (Spitzer, 1956).

$$\lambda \sim T^2 / n$$

Fluid limits

(Howes, 2024)

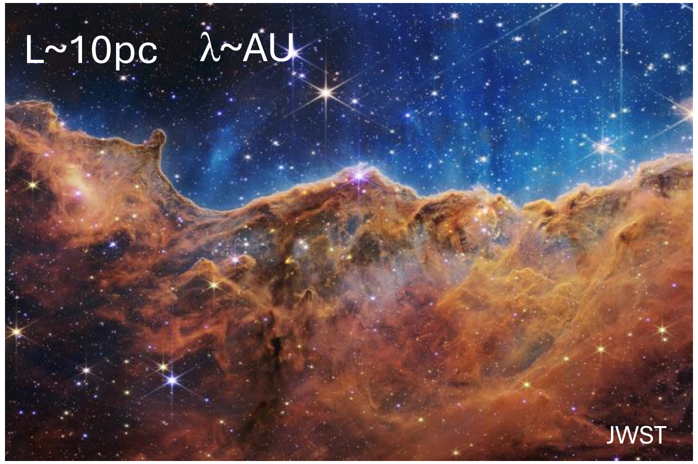
Mean Free Path/L

MFP << Scale:

Fluid works as approximation.

Meteorology

Giant Molecular Cloud



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(Howes, 2024)

Mean Free Path/L

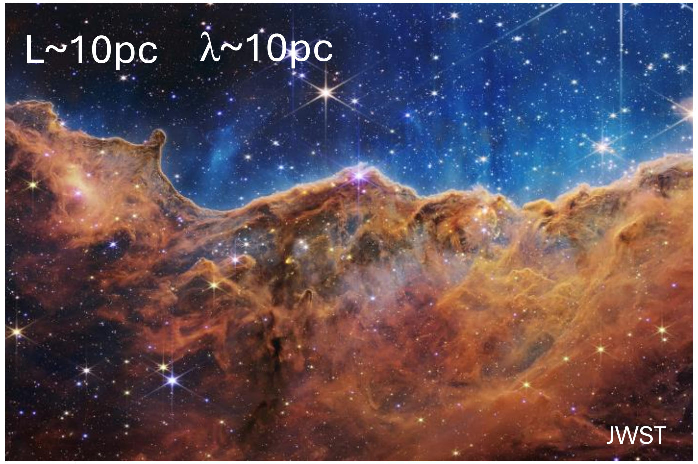
MFP << Scale:
Fluid works as approximation.

MFP ~ Scale:
Requires higher order corrections.

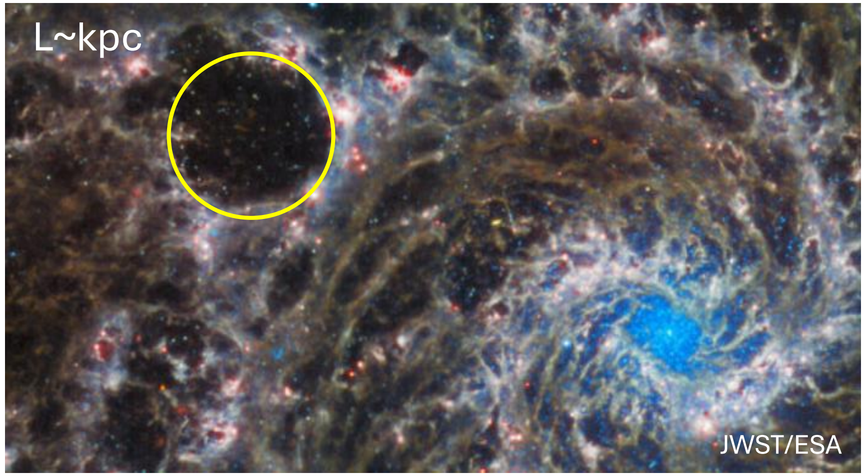
Meteorology

Solar Corona

Giant Molecular Cloud



Superbubbles in Galaxies



Fluid limits

(Howes, 2024)

Mean Free Path/L

MFP << Scale:
Fluid works as approximation.

MFP ~ Scale:
Requires higher order corrections.

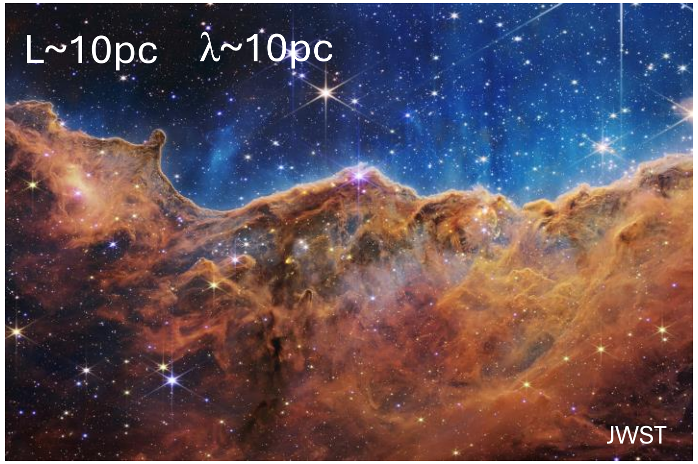
MFP > Scale:
Fluid does not work at all.

Meteorology

Solar Corona

Aurora Borealis

Giant Molecular Cloud



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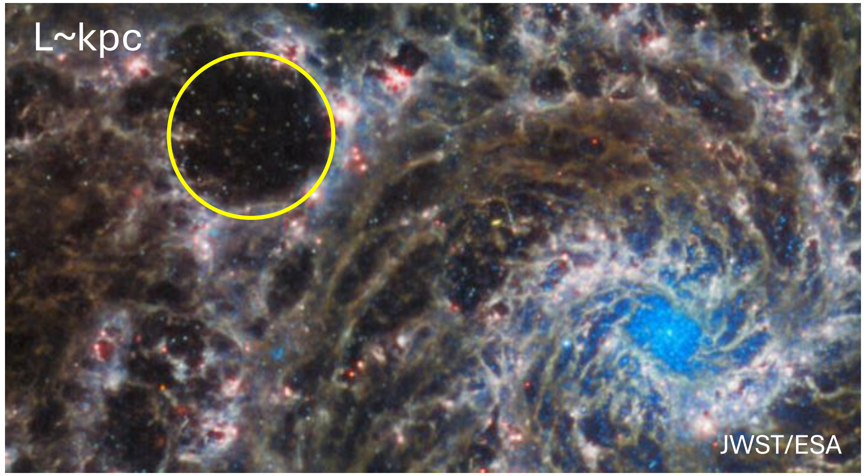
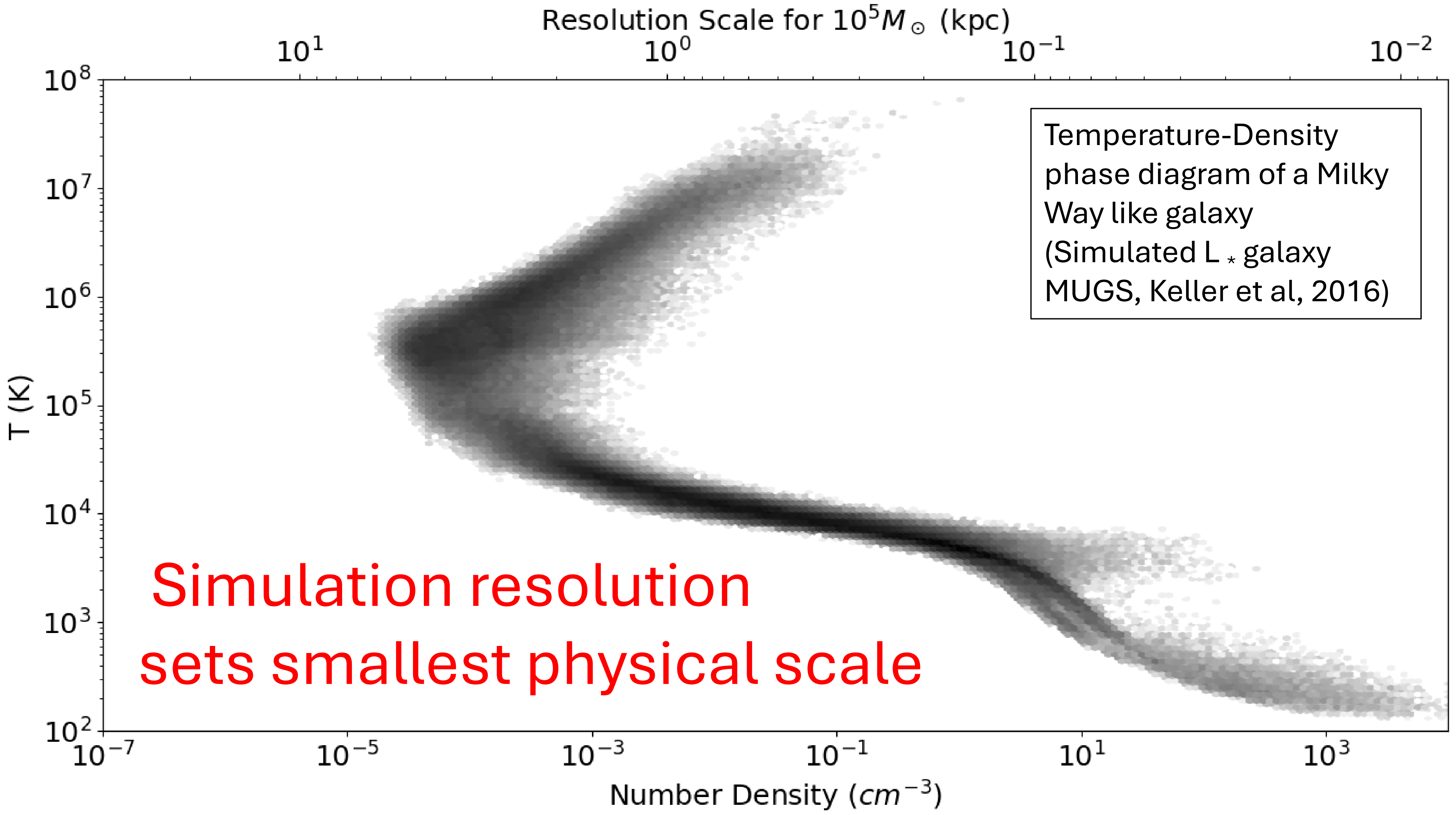
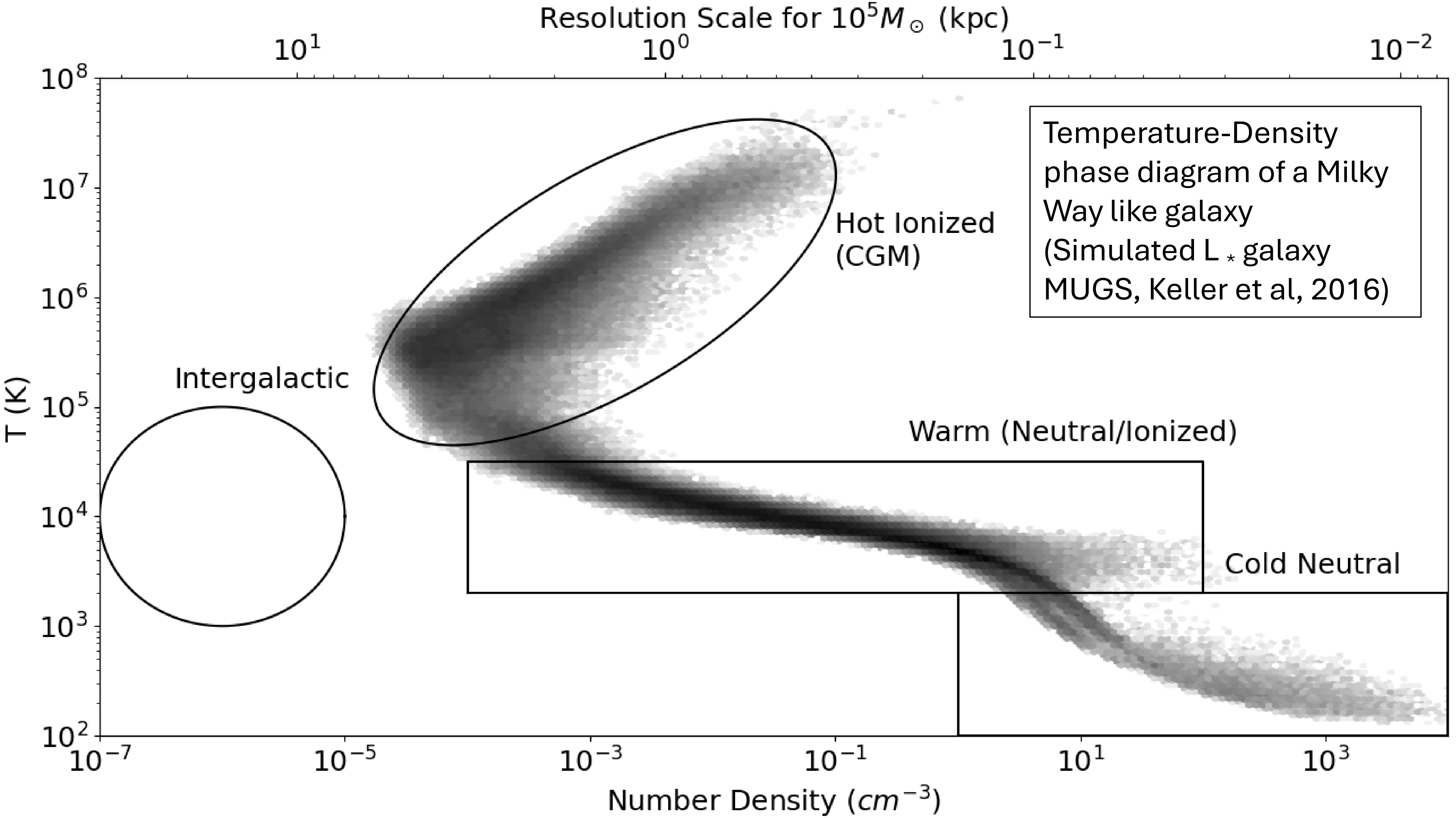


Intra Cluster Medium

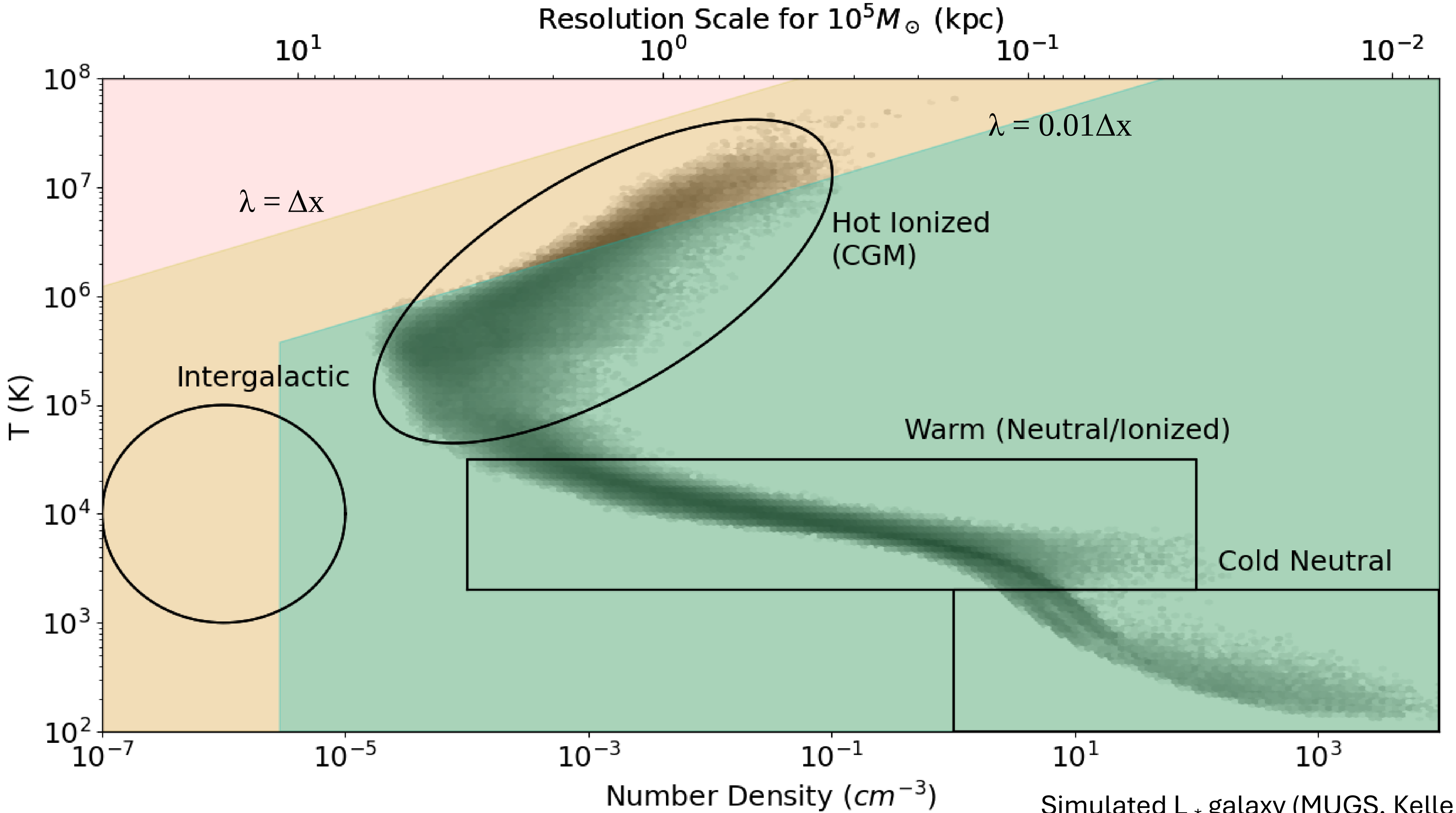
Galaxy Simulations: Mean Free Path vs. Resolution



Galaxy Simulations: Mean Free Path vs. Resolution

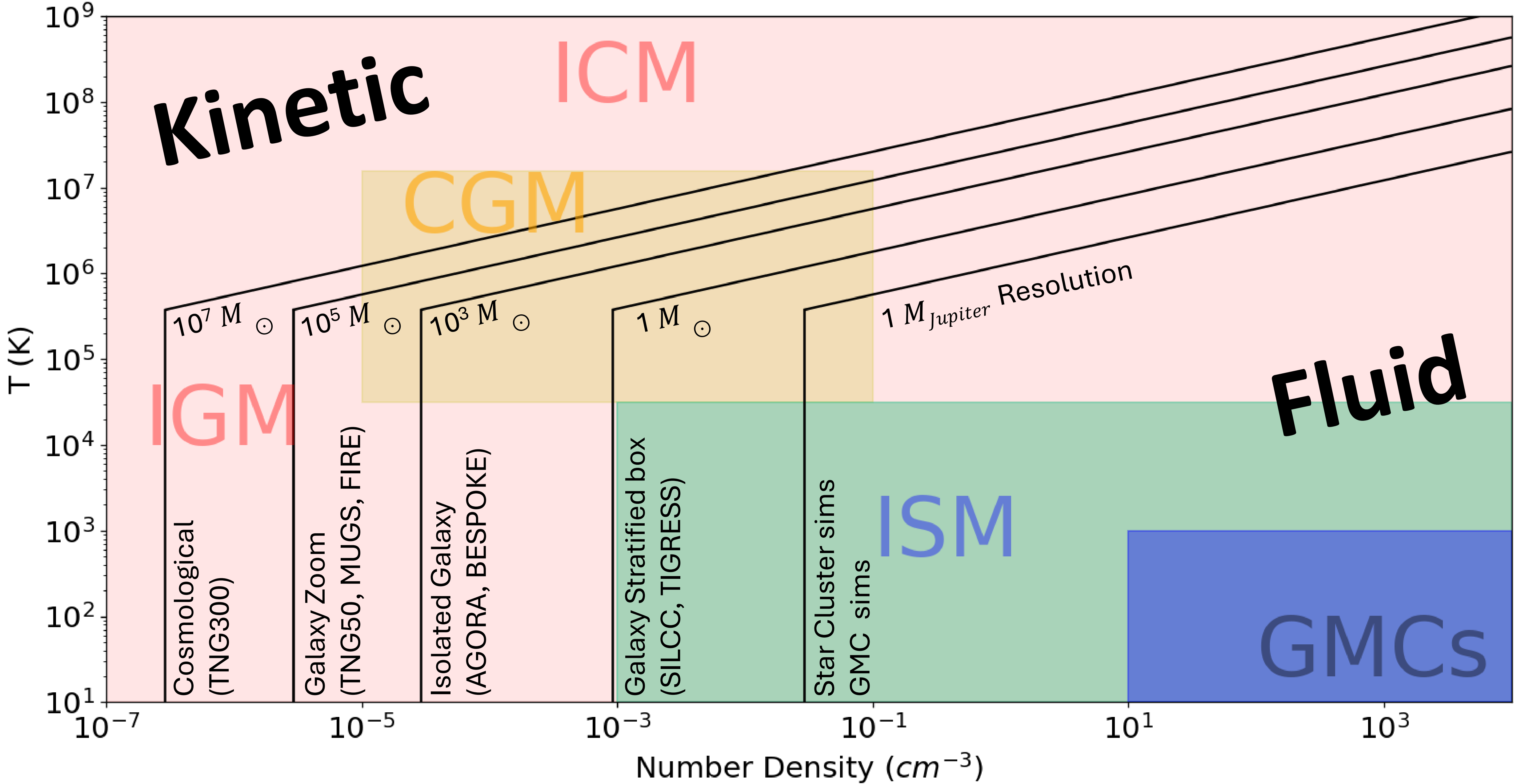


Green: Resolution >> Mean free path: Fluid: Euler is good
Yellow: Resolution ~ Mean free path: need corrections (e.g. Conduction)
Red: Resolution << Mean free path: Kinetic Theory



Simulated L_* galaxy (MUGS, Keller et al. 2018)

General Simulations: Fluid / Kinetic regimes vs. resolution



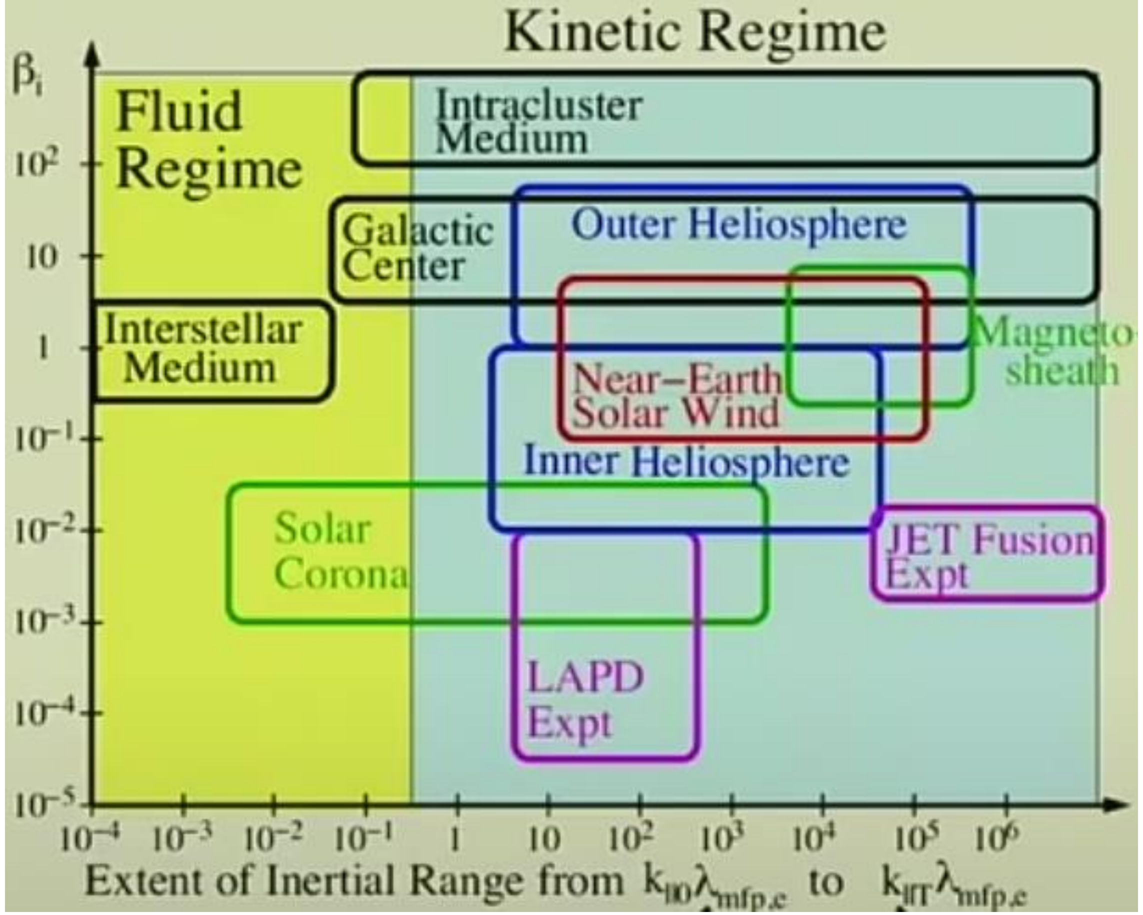
Magnetized fluids also suffer from kinetic effects

Some solar cosmic ray simulations have 8th order (!) corrections

(Howes, 2024)

Magnetic fields related to density via broken power law (Whittworth, 2024, in review)

β = Thermal Pressure/Magnetic Pressure



Mean free path is on the x axis, versus magnetic field β on the y (Howes, 2024) .

Green: Resolution >> Mean free path: Fluid: Euler is good
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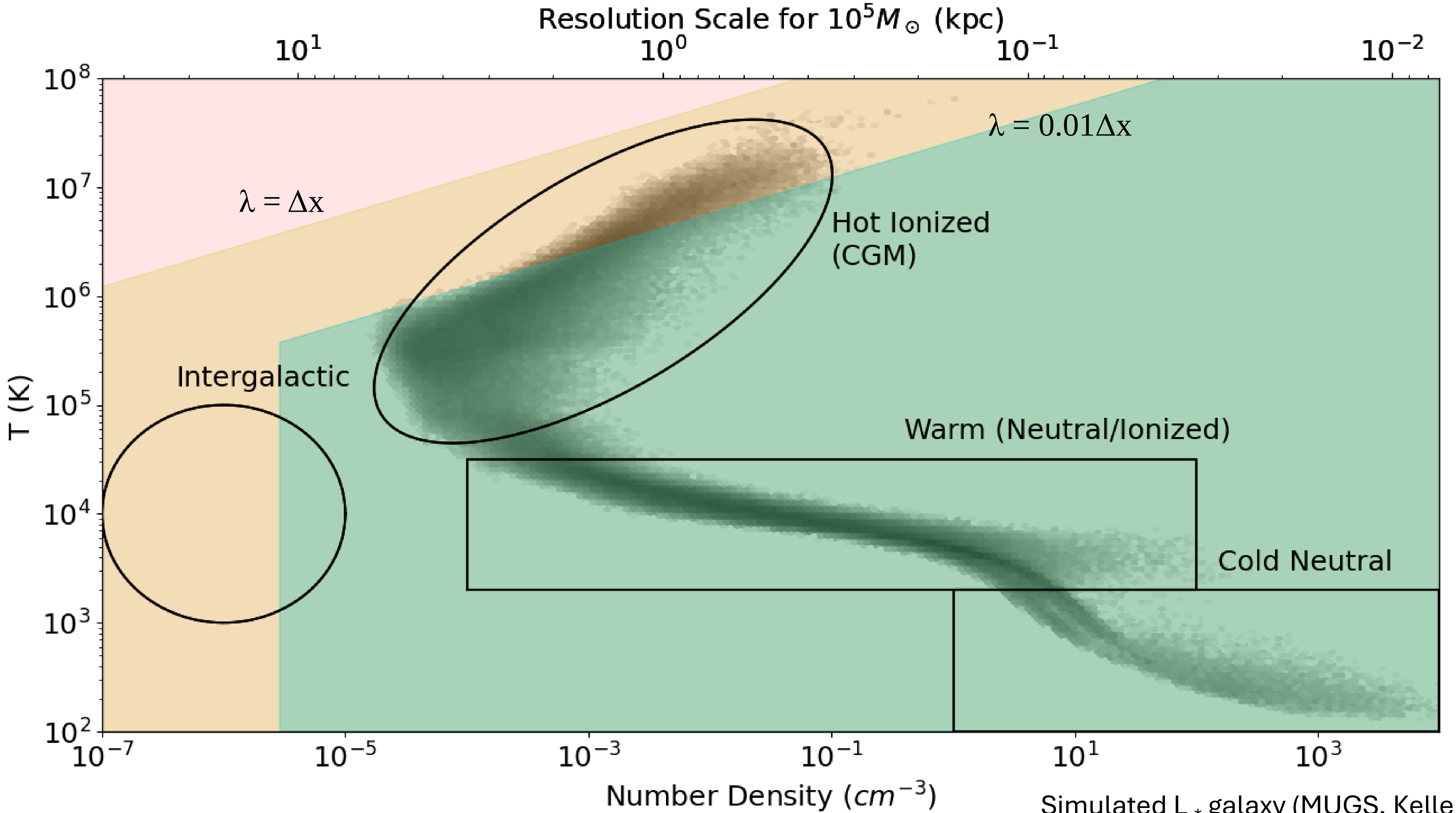
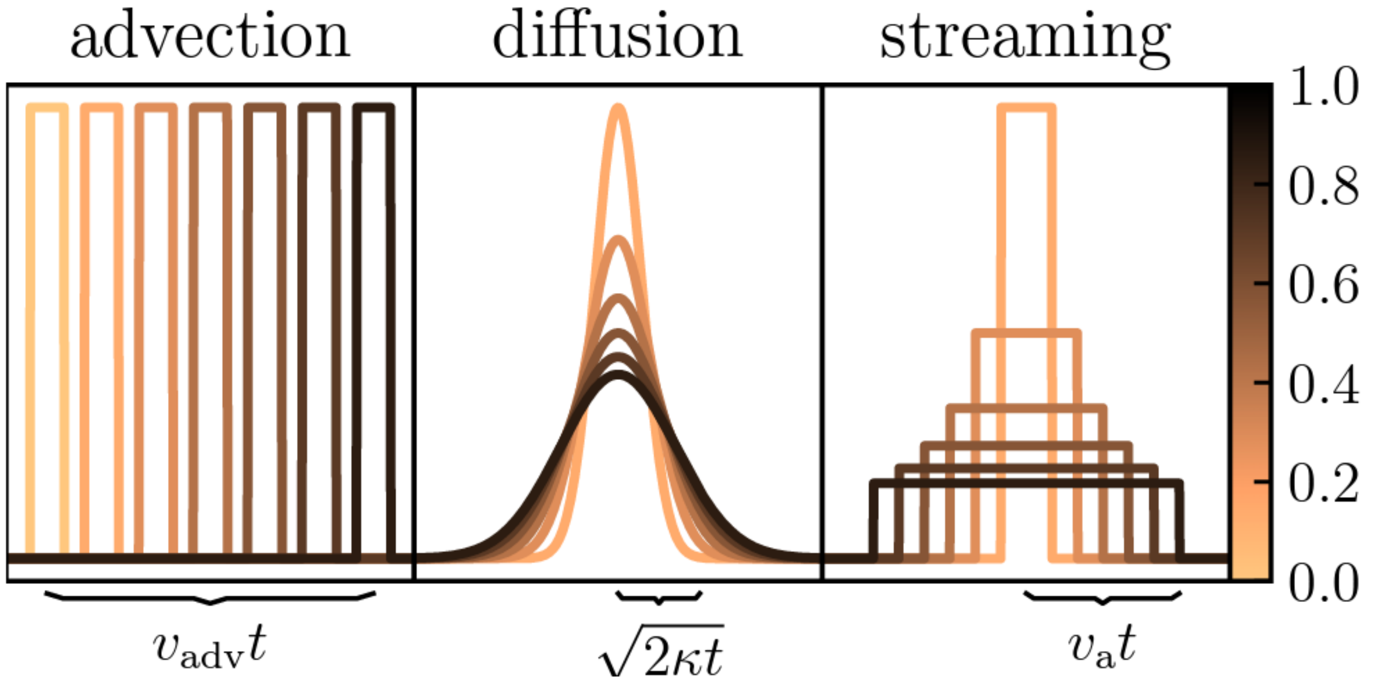


Simulated L_* galaxy (MUGS, Keller et al. 2018)

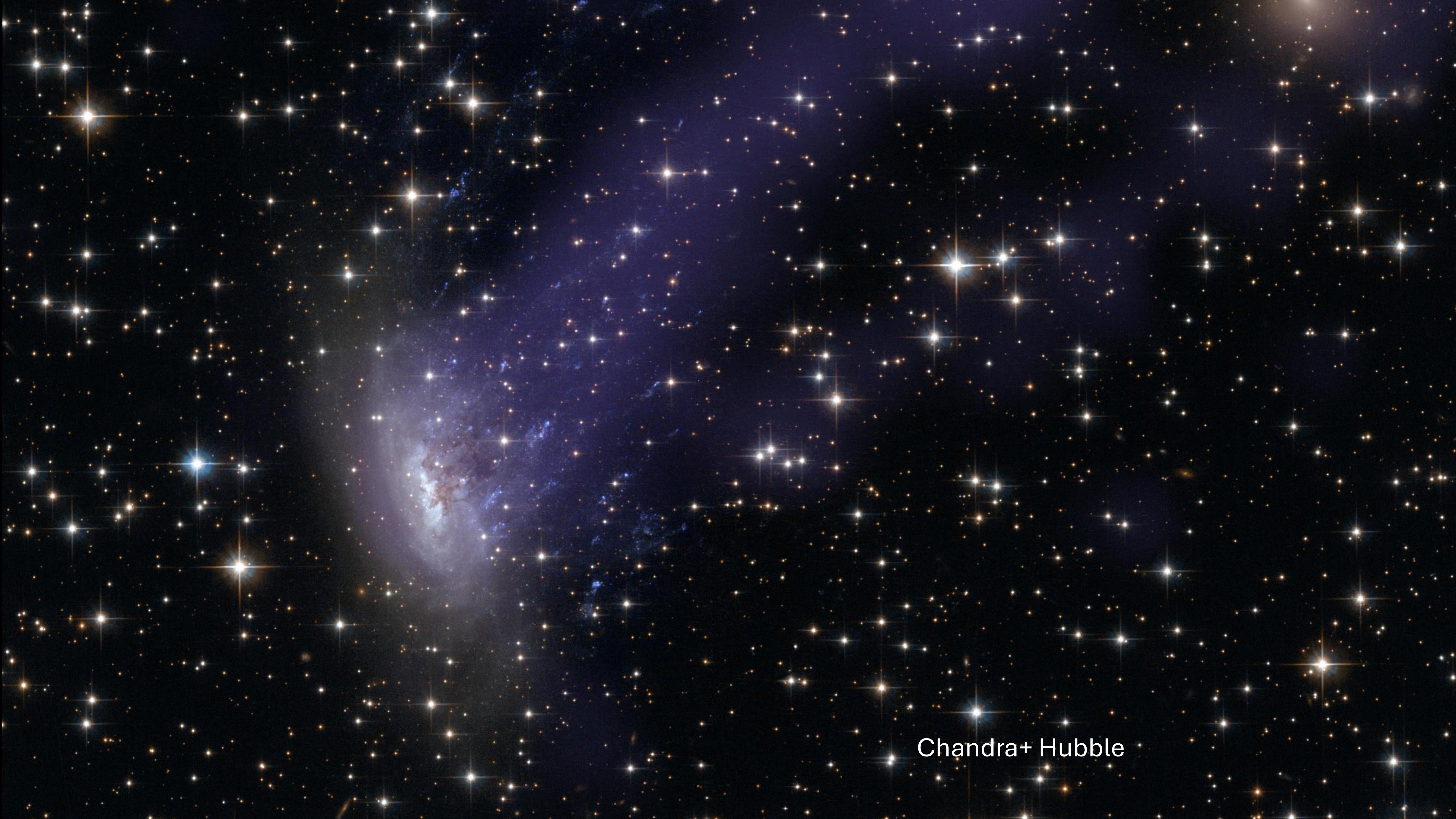
Cosmic Rays as a Fluid

Evolving cosmic ray as a fluid

- Can advect with the gas.
- Can diffuse along magnetic field lines.
- Can stream along field lines

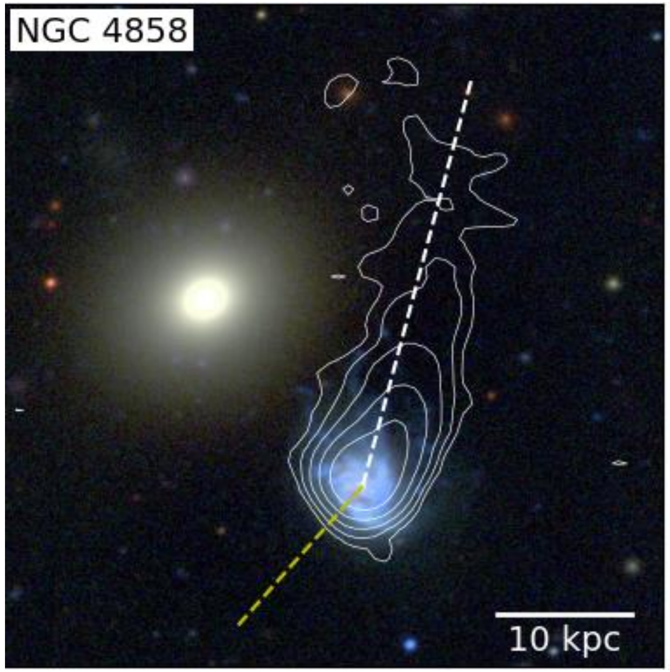


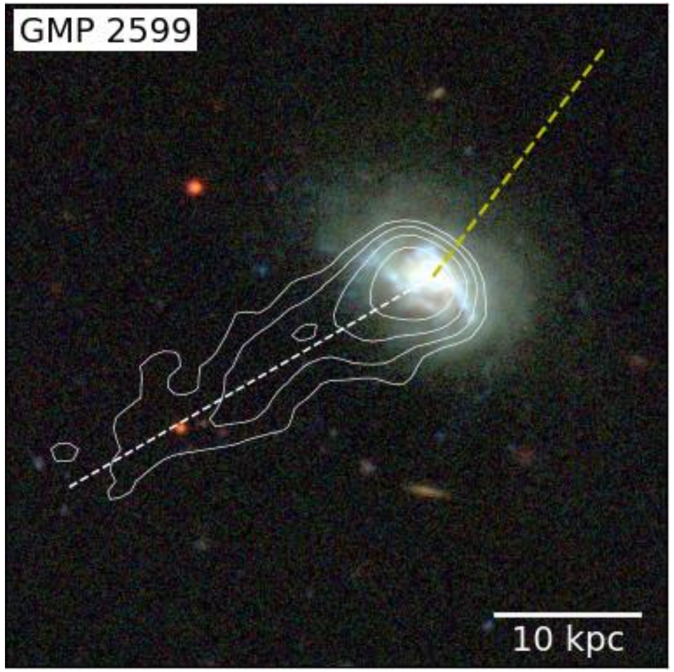
Ruszkowski and Pfrommer (2023)


Chandra+ Hubble

Current Work: Cosmic Rays in Galaxies

- Cosmic rays generated by supenovae/superbubbles
- Transport depends on the local magnetic field (Zweibel, 2013).
- Equations for hyperbolic conduction can be adapted to model cosmic rays (Chan, 2019)
- Cosmic rays observed in the tails of jellyfish galaxies with LOFAR (Roberts et al 2023):
  Potential for constraining cosmic rays & magnetic fields





Roberts et al, 2024

Modeling Diffusion and Streaming

Parabolic (standard moment)

- Follows the standard heat equation
- Allows propagation at infinite speed
- Ad hoc saturated flux cap
- Computationally slow: speed ~ resolution^2

$$\mathbf{F}_{\rm di} \; = \; -\kappa\,\hat{\mathbf{B}}\,(\hat{\mathbf{B}}\,\cdot\,\nabla e_{\rm cr})$$

Hyperbolic Equations

- Alternative to standard heat equation.
- Heat transfers at a finite speed (Axford, 1965)
- No need for ad hoc saturated flux cap
- Computationally fast: speed ~ resolution
- Implemented in Gasoline2 (SPH code)

$$\frac{1}{\tilde{c}^2}\left[\frac{\partial\tilde{\mathbf{F}}_{\rm cr}}{\partial t}+\nabla\cdot\left(\mathbf{v}\otimes\tilde{\mathbf{F}}_{\rm cr}\right)\right]+\nabla_{\parallel}P_{\rm cr}=-\frac{(\gamma_{\rm cr}-1)}{\kappa^*}\tilde{\mathbf{F}}_{\rm cr}$$

Chan et al 2019

Modeling Diffusion and Streaming

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$$\frac{\partial}{\partial t} T = \kappa \nabla^2 T$$

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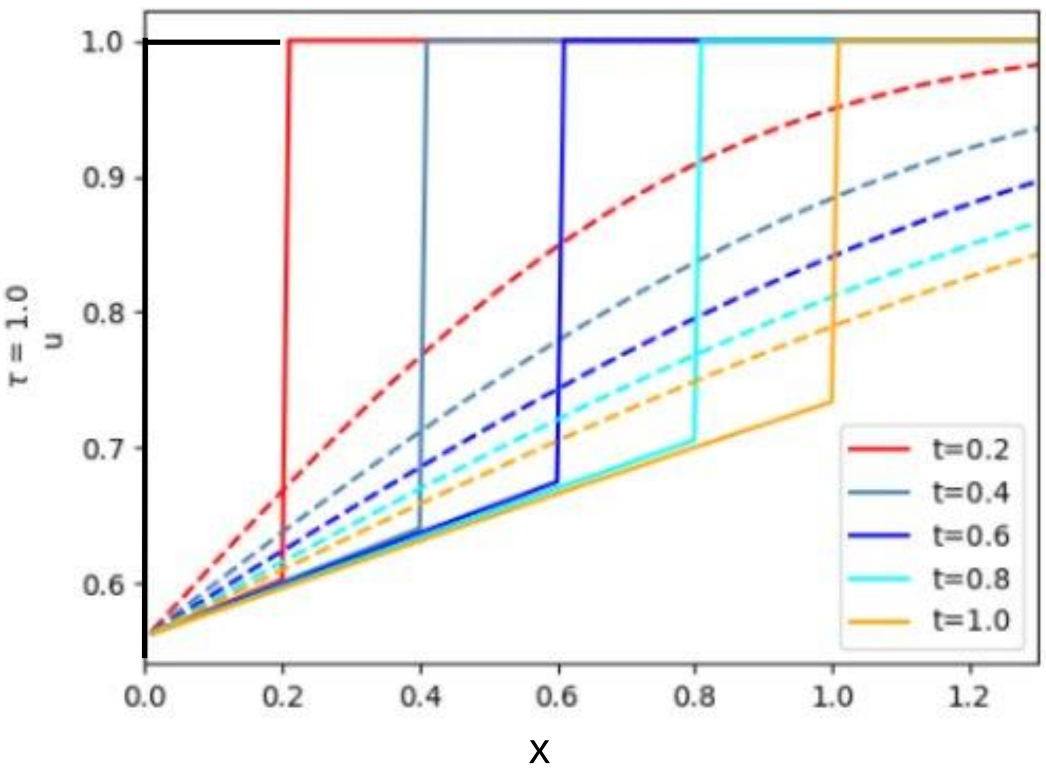
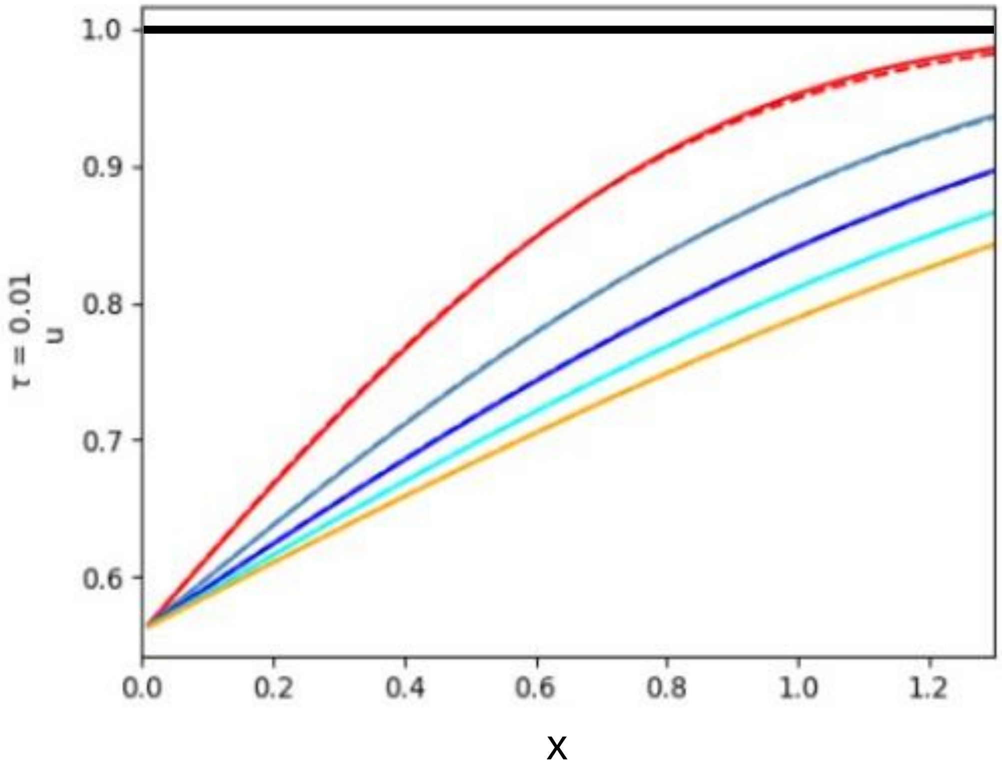
$$\frac{\partial^2}{\partial t^2} T + \frac{1}{\tau} \frac{\partial}{\partial t} T = \frac{\kappa}{\tau} \nabla^2 T$$

Owens & Wadsley, 2024

Hyperbolic vs. Traditional diffusion

- Test problem: Step at x=0, t=0 (black line) (Owens & Wadsley, 2024)
- Dashed lines are traditional conduction, solid are hyperbolic
- Information propagates at speed of carriers (e.g. electrons in plasma)

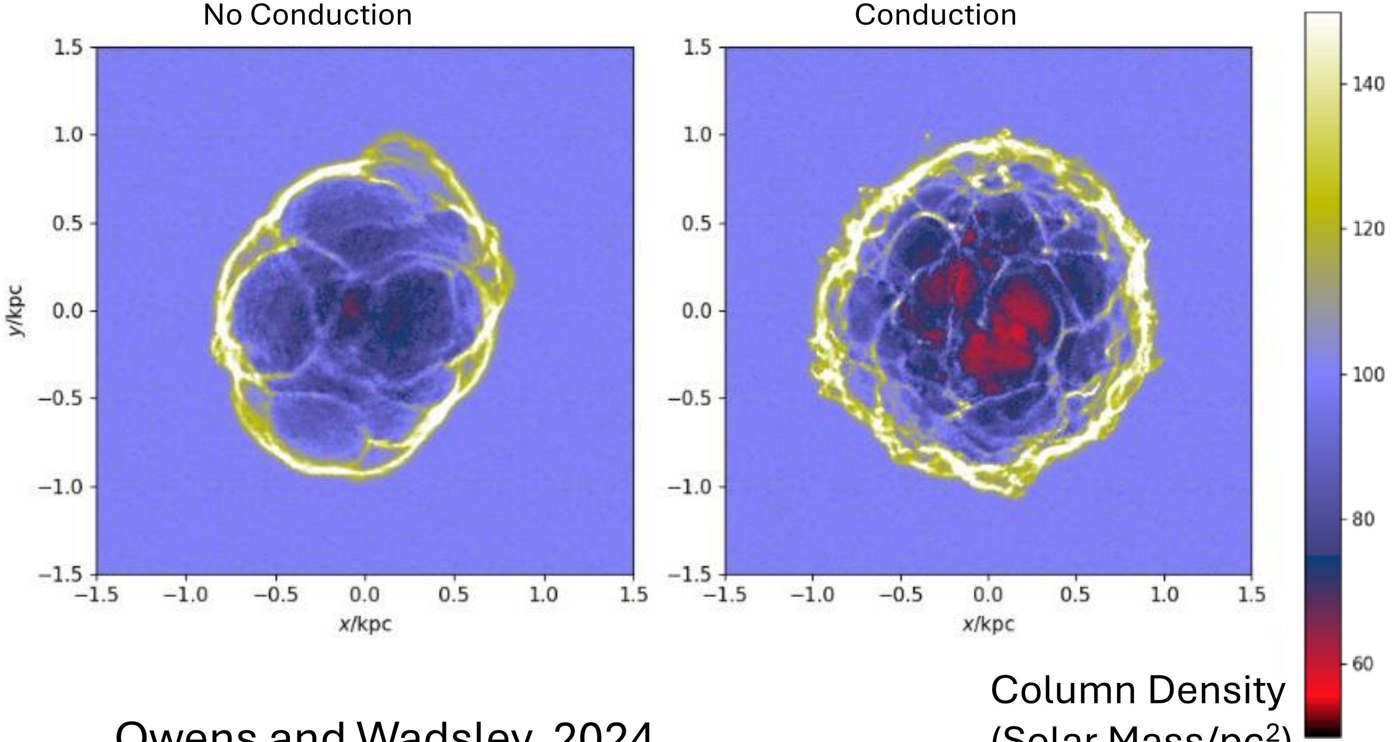




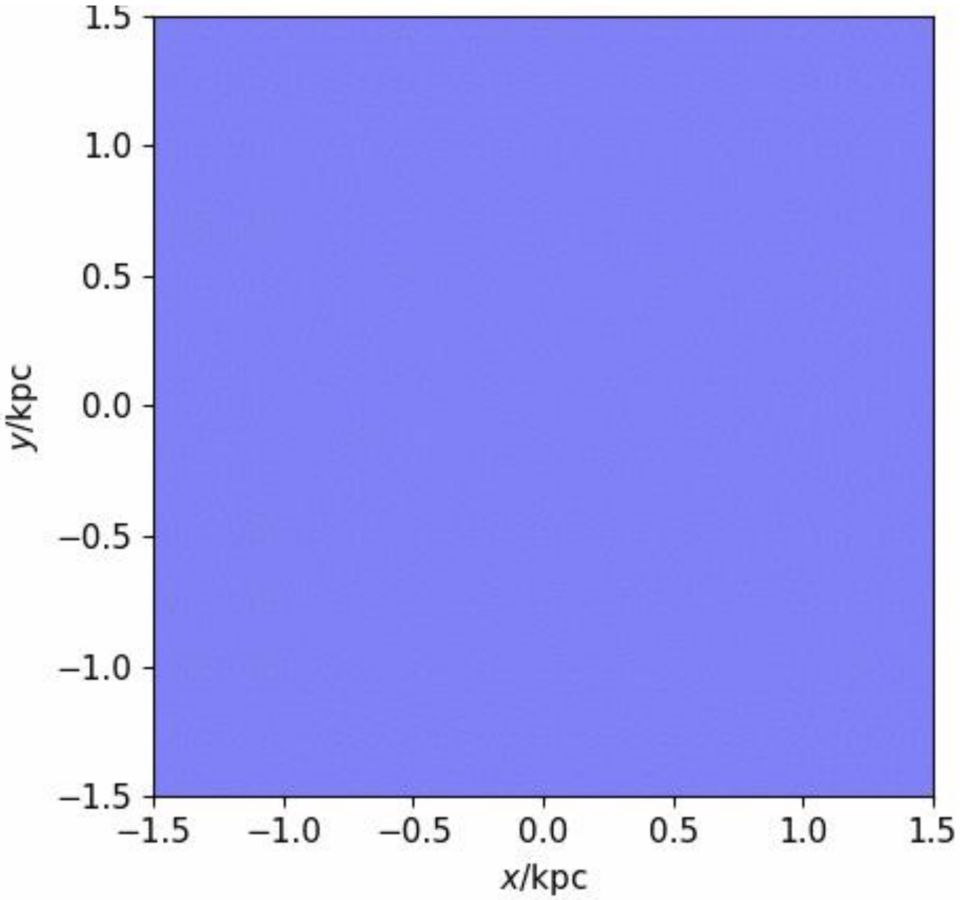
Superbubbles: Euler vs. Spitzer Conduction

Conductive bubble:

- Radius about 10% larger
- Six times the hot mass
- Temperature regulated by conductions to ~ 2 x 10^6 K (order of magnitude lower)



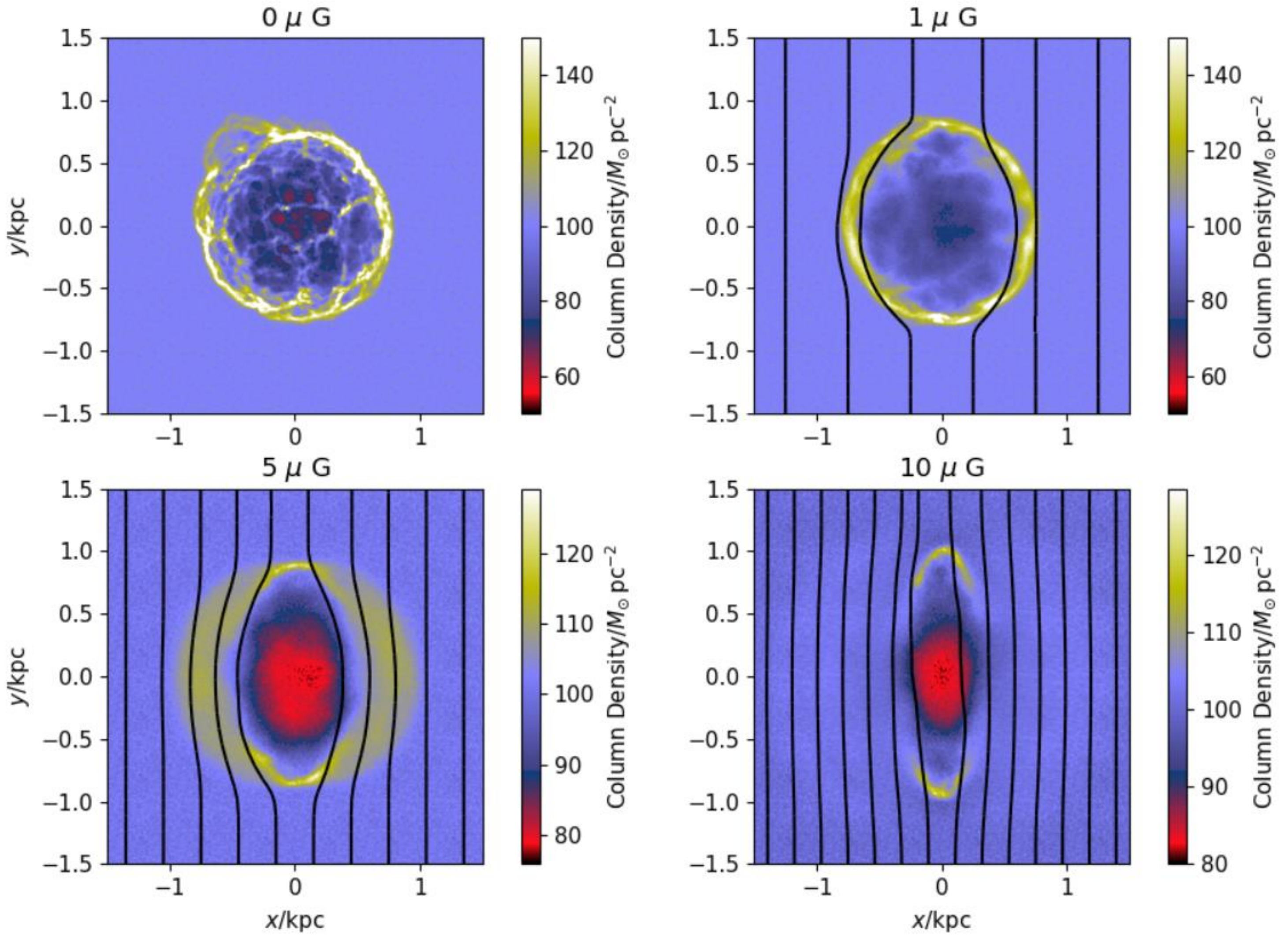
Column Density (Solar Mass/pc^2)



Owens and Wadsley, 2024

Superbubbles in magnetised gas

- Magnetic pressure comparable to thermal, turbulent pressure in galaxies/ISM
- Stronger magnetic fields result in an under-sized, anisotropic, bubble.
- Magnetic fields can have a strong influence over galactic superbubbles, e.g. sizes and lifetimes (Hix et al. 2023; Robinson & Wadsley, 2024).



Superbubbles in initially uniform magnetic fields
(Owens, Wissing & Wadsley, in prep)

Tests performed with Gasoline2-MHD (Wissing et al, 2020).

Summary

- The mean free path should be taken into account when using a fluid-like model.
- Hyperbolic conduction is more physical and computationally faster compared to traditional conduction.
- Our model of hyperbolic conduction displays the correct behaviour for high temperatures (> 1 000 000 K), including MHD
- Cosmic Rays potentially important galactic feedback as well