Magnetohydrostatic Structures of Magnetically-Supported Filaments and their Stability

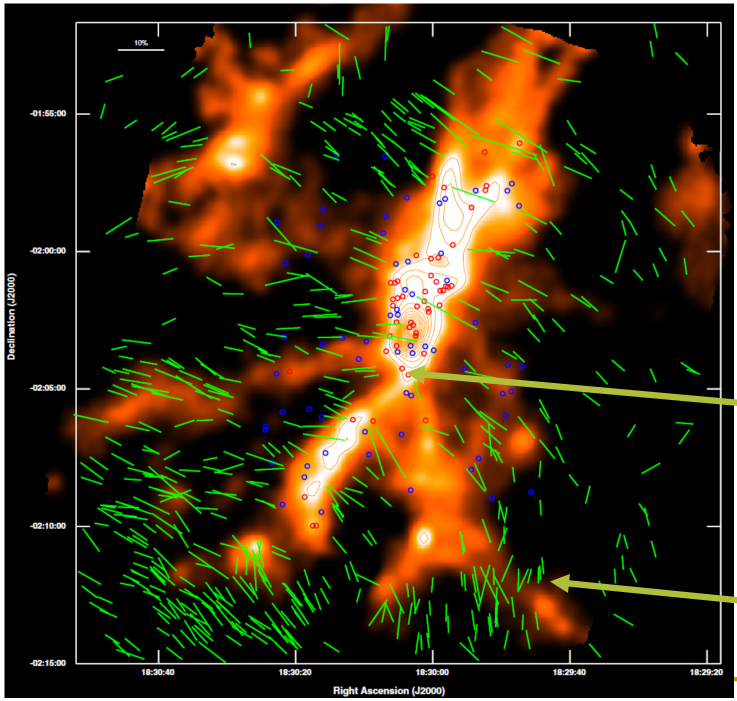
Kohji Tomisaka（National Astronomical Observatory of Japan）

1) Structure and Mass of Filamentary Isothermal Cloud Threaded by Lateral Magnetic Field, 2014, ApJ, 785, 24(12pp)

2) Polarization Structure of Filamentary Clouds, 2015, ApJ, 807, 47(10pp) Jul.1

Filamentary Cloud

- *Herschel* has revealed many filaments in thermal dust emissions. Filaments are regarded as basic building blocks of clouds.
- Near IR polarization extinction observations indicate
  - Interstellar magnetic field is ⊥ to the filaments with large column-density.
  - low column-density filament is extending // to B.

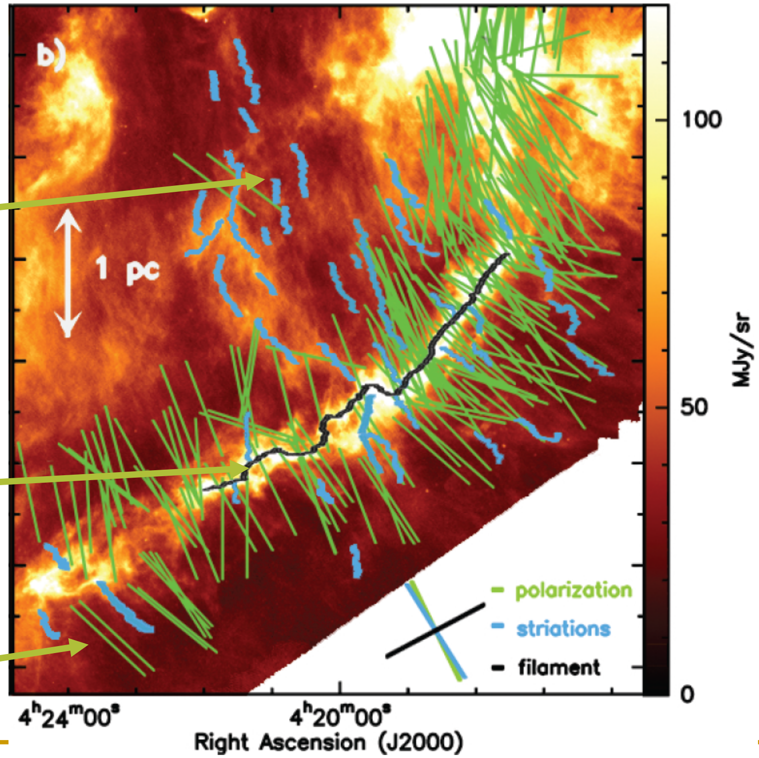


Serpens South Cloud by Sugitani et al (2011).

Less-dense filaments with small Σ

Dense filaments with large Σ

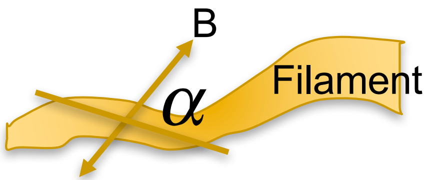
Pol E-vector IS B-field



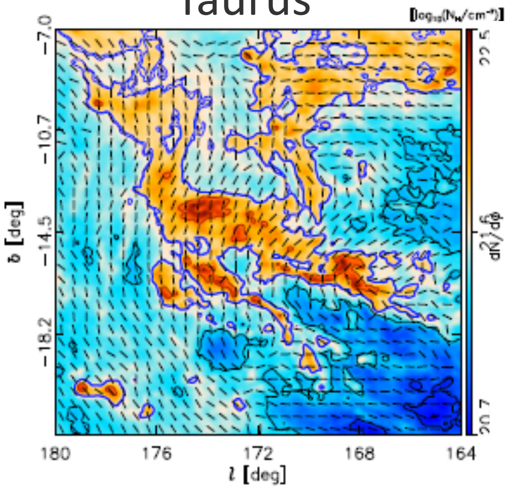
Taurus Cloud (B211/213) by Palmeirim et al. (2013).

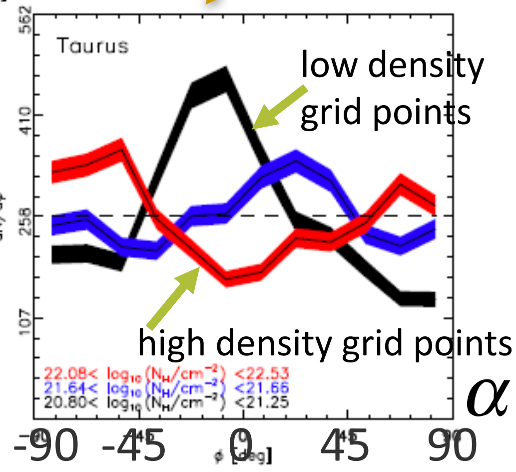
Planck Polarization (353GHz)

Planck intermediate results. XXXV (2015). Polarization of Thermal Dust Emission



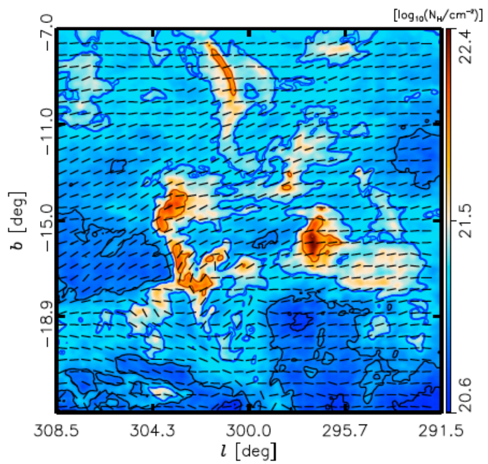
Taurus

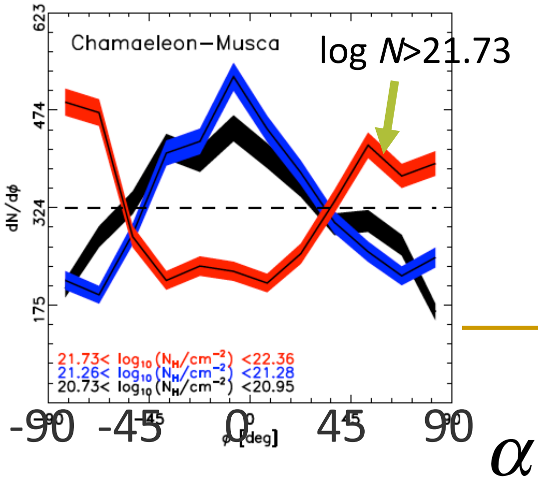




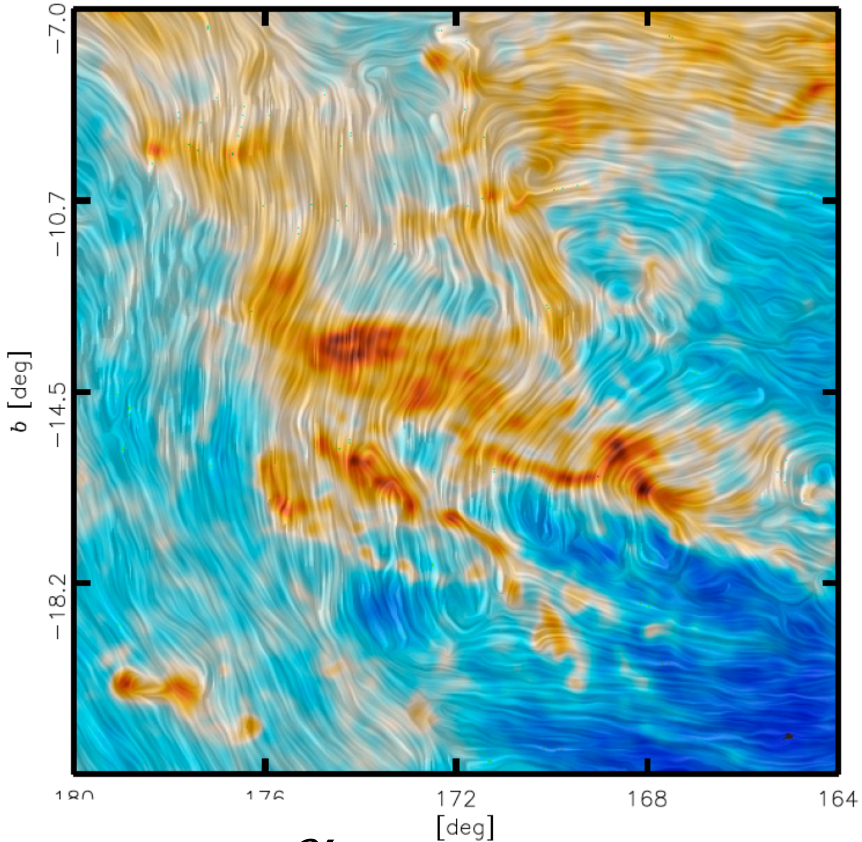
➔ α~90 deg for high-density portion log *N*>22.08

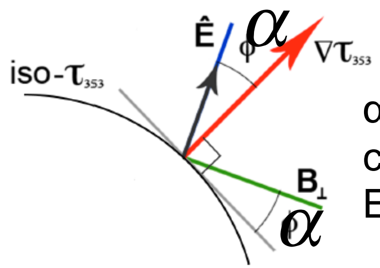
Chamaeleon-Musca





Taurus





α is an angle b/w density contour line and B-vector of EM wave

high density structure prefers α ~90 deg.
But Universal?

B-Field plays a Role in Stability of the Filament?

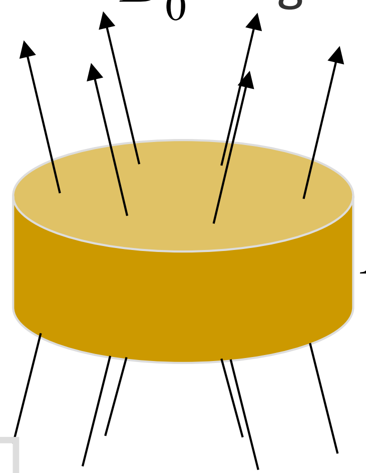
- Stability is controlled by magnetic flux.

Critical Mass $M_{\rm cr} \approx \Phi_{\rm 2D} / 2\pi G^{1/2}$

Magnetic Flux $\Phi_{\rm 2D} \equiv \int \mathbf{B} \cdot d\mathbf{S} = \pi R_{cl}^2 B_0$

$M / \Phi_{\rm 2D} > M_{\rm cr} / \Phi_{\rm 2D} = 1 / 2\pi G^{1/2}$

- Clouds with $M > M_{\rm cr}$ supercritical
  - They have no static equilibrium.
  - Dynamical contraction
- Clouds with $M < M_{\rm cr}$ subcritical
  - Hydrostatic equilibria
  - Quasi-static contraction driven by ambipolar diffusion
- How about filamentary clouds?

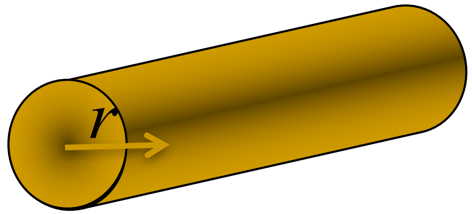


Equilibria of Isothermal filamentary Clouds

- No Magnetic Field (Stodolkiewicz 1963; Ostriker 1964)

$$\rho(r) = \rho_c \left(1 + \frac{r^2}{8H^2}\right)^{-2}$$

Scale-height $H = c_s / (4\pi G \rho_c)^{1/2}$



- Line-mass [g/cm, $M_\odot$/pc]

$$\lambda(R) \equiv \int_0^R 2\pi r \rho(r)\, dr = \frac{2c_s^2}{G} \frac{R^2/8H^2}{1 + R^2/8H^2} \le \frac{2c_s^2}{G} \quad (r \gg H)$$

- Max. line-mass

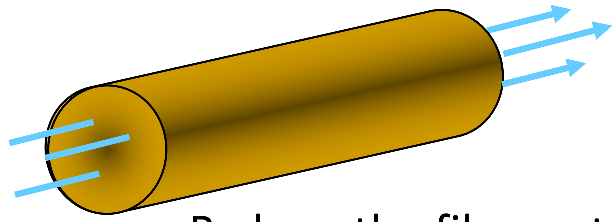
$$\lambda_{\max} = \frac{2c_s^2}{G}$$

critical line-mass of B=0 case

$\lambda > \lambda_{\max}$ ➔ No equilibria ➔ dyn. contraction

$\lambda < \lambda_{\max}$ ➔ equilibrium solution with a finite density-contrast

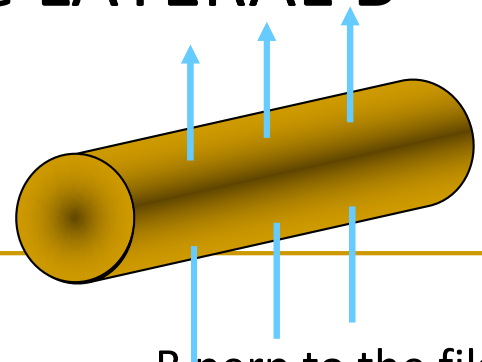
Magnetized Filaments



- Model with constant plasma β
  $(\beta \equiv p / (B_z^2 / 8\pi))$ (Stodolkiewicz 1963)

$$\lambda = \frac{2c_s^2}{G}(1+\beta^{-1})\frac{R^2/8H^2}{1+R^2/8H^2} \qquad H = \frac{c_s(1+\beta^{-1})}{(4\pi G\rho_c)^{1/2}}$$

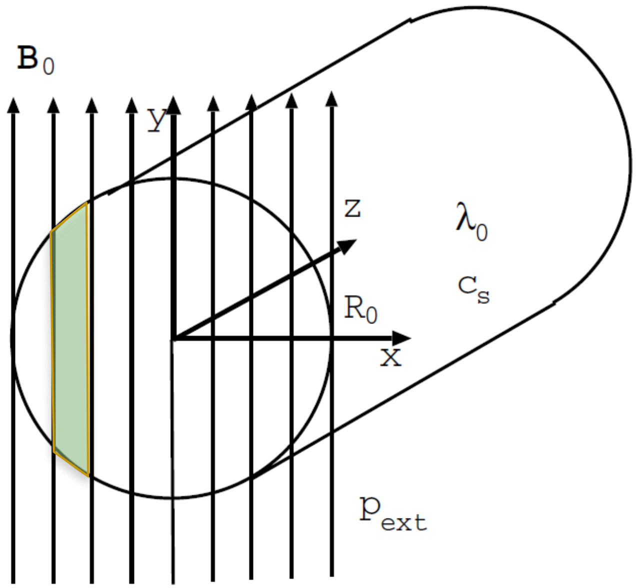
- Model with a constant mass/flux ratio
  （ $\phi \equiv \rho / B_z$ is conserved in the radial contraction）
  (Fiege & Pudritz 2000a,b)
  - Line-mass increases with B-field strength.
- However, observed filaments have LATERAL B-field.



Parameters to Specify a Magnetohydrostatic Equilibrium

"Parent" filament

defines a way of mass-loading

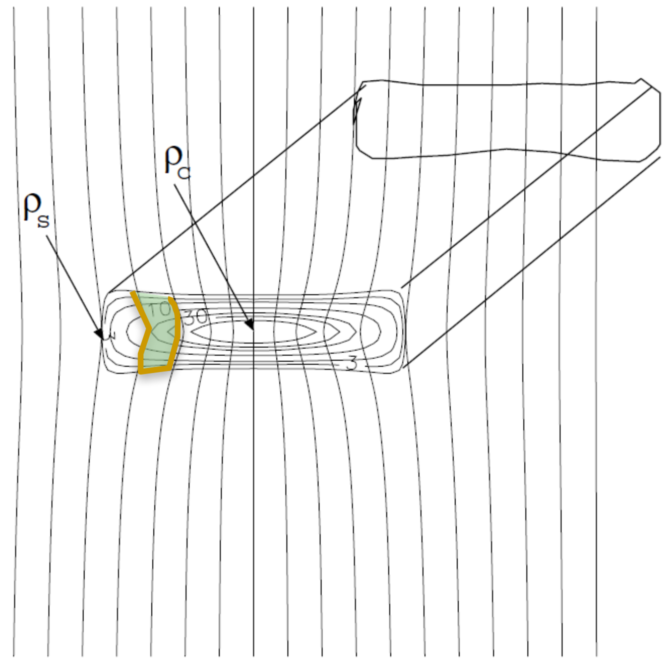


We consider a gas cylinder with a uniform density, a radius R_0, and sound speed c_s is immersed in a uniform B-field B_0 and external pressure p_{ext}.

Flux freezing

Equilibrium

in balance b/w gravity, Lorentz force, and thermal pressure



Thin and wide noodle

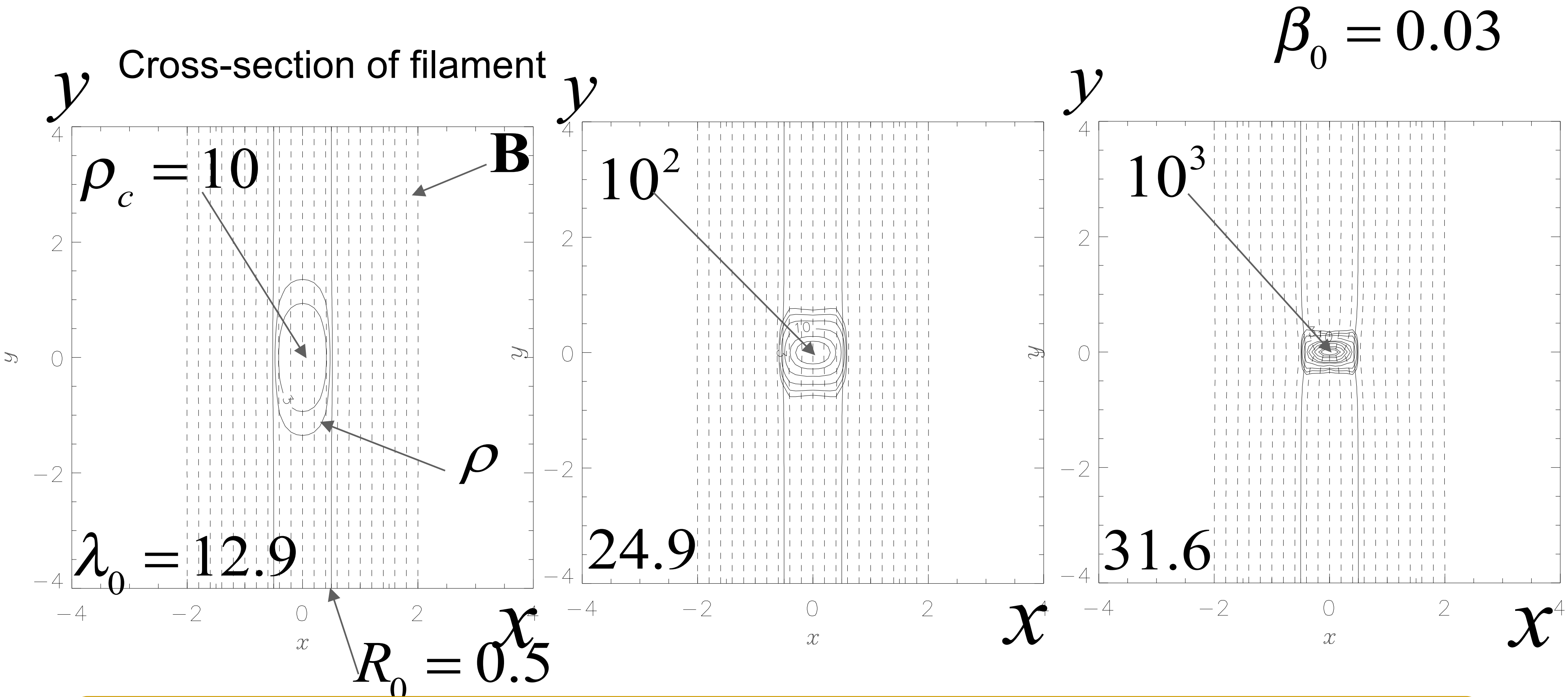
density at the surface $\rho_s = p_{ext} / c_s^2$

central density ρ_c

After normalization, the problem contains 3 parameters:

Density contrast	Ambient plasma β	Radius of "Parent" filament
ρ_c / ρ_s	$\beta_0 \equiv p_{ext} / (B_0^2 / 8\pi)$	$R_0 / [c_s / (4\pi G \rho_s)^{1/2}]$

Result 1 Small R_0=0.5 of Parent Cloud



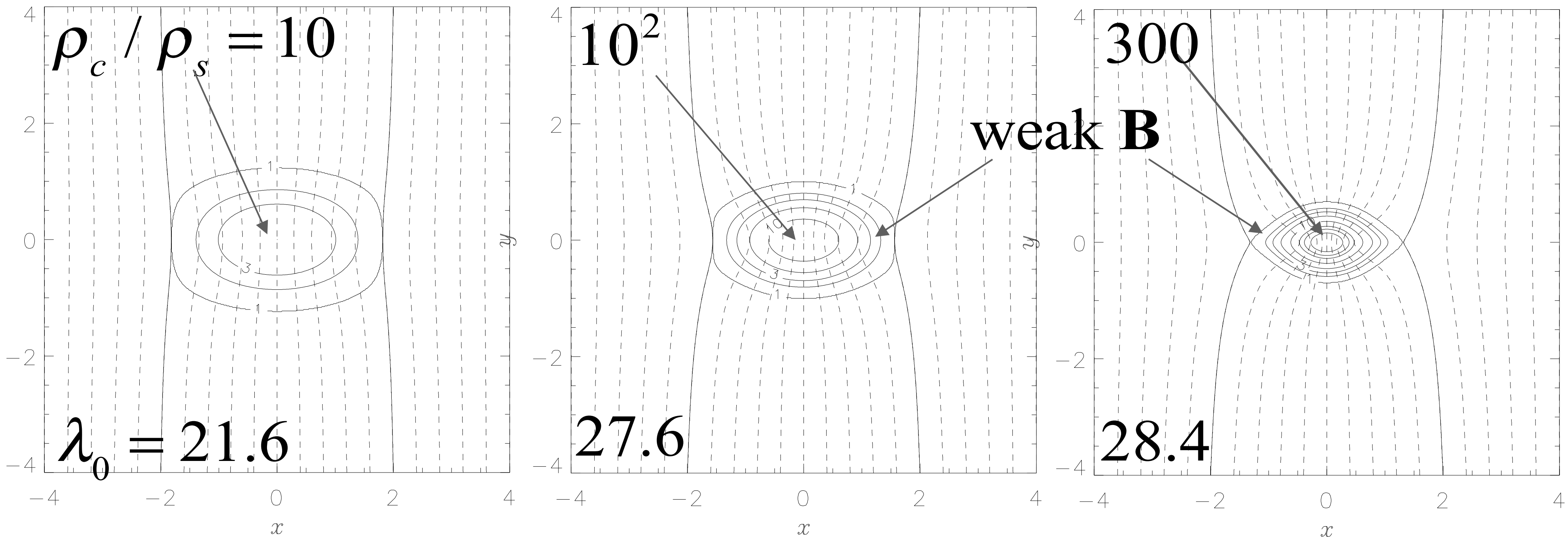
(1) Line-mass λ_0 increases with central density ρ_c.

(2) The filament with low ρ_c extends along B-field.

(3) That with high ρ_c has a major axis perp to B-field.

Result（2） Standard Model

$R_0 = 2,\ \beta_0 = 1$

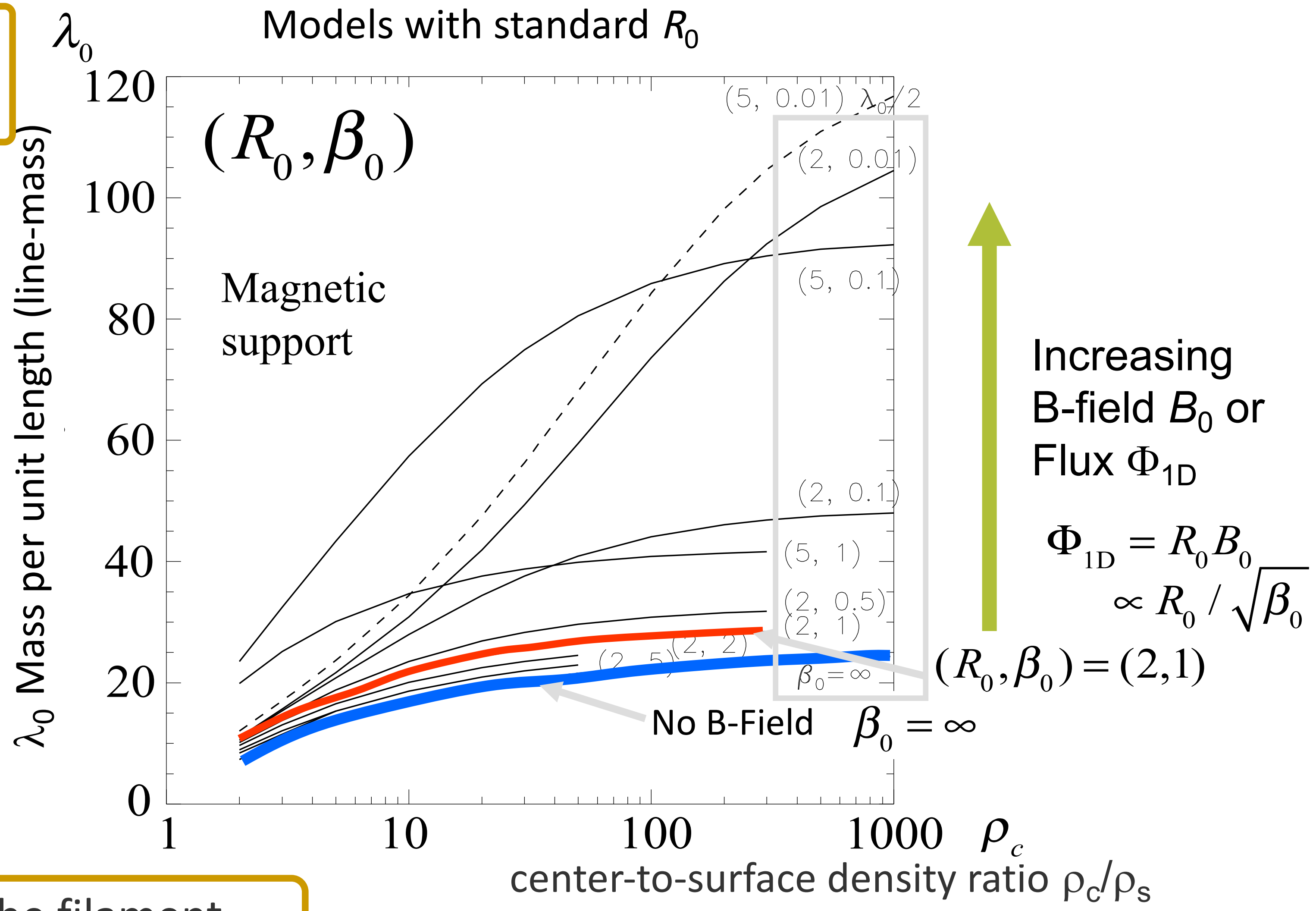


Hour-glass type B-field.

(1) Line-mass λ_0 increases with central density ρ_c.
(2) The major axis is perpendicular to B-field.
(3) Regions of weak B-field are found near the equator.

Central Density ρ_c vs Line-Mass λ_0 Relation

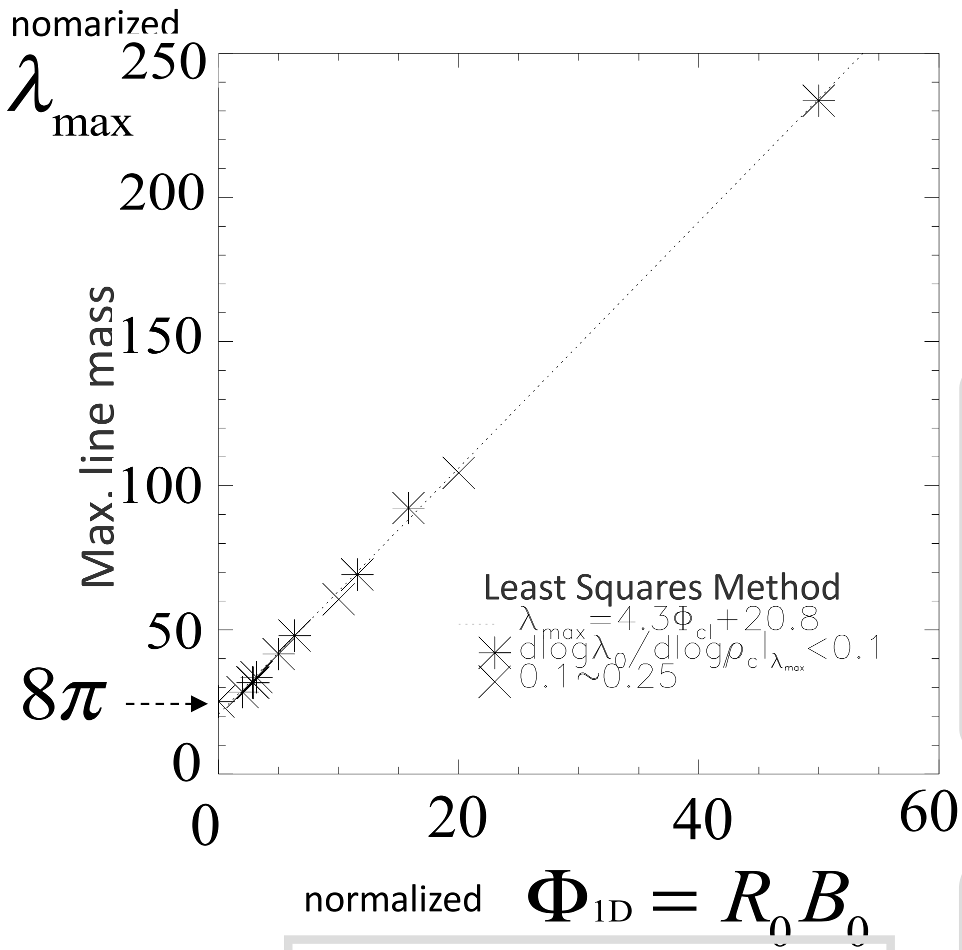
Models with not too small R_0



B-field supports the filament

Maximum mass supported by a given Φ_{1D} is achieved at $\rho_c/\rho_s=\infty$.

Critical Line-Mass of the Filament



Mag. flux per unit length

Empirical critical mass formula

$$\lambda_{max} \approx 0.24\Phi_{1D} / G^{1/2} + 1.66 c_s^2 / G$$

dimensional

When the magnetic flux exceeds

$$\Phi_{1D} = R_0 B_0 > 3\mu\text{G pc}$$

maximum line-mass is determined by the magnetic flux per length.

Take notice of the similarity to the mass formula for a thin disk

$$M_{max} \approx \Phi_{2D} / 2\pi G^{1/2} \cong 0.16\Phi_{2D} / G^{1/2}$$

Polarization of Thermal Dust Emissions from oblate/prolate dusts aligned in the B-field direction.

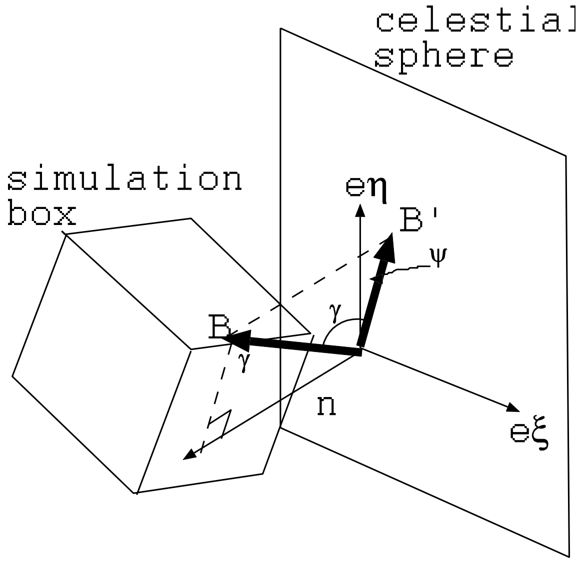
$$Q = \int C \cdot R \cdot F \cdot c \cdot B_v(T) \rho \cos 2\psi \cos^2 \gamma \, ds$$
$$U = \int C \cdot R \cdot F \cdot c \cdot B_v(T) \rho \sin 2\psi \cos^2 \gamma \, ds$$

(Draine & Lee 85, Fiege & Pudritz 2000)

C: difference of cross sections perp and parallel to B
R: reduction factor due to imperfect grain alignment
F: reduction factor due to turbulent B-field

$c = \rho / n_d$

γ : angle b/w B and plane of the sky.
ψ : angle b/w projection of B and η-axis

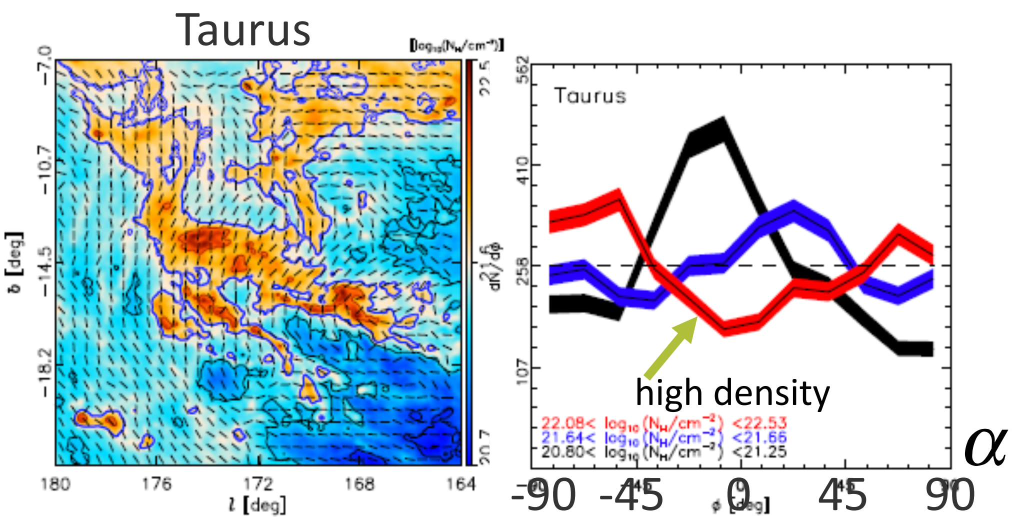


Relative Stokes parameter (Wardle & Konigl 90)

$$q = \int \rho \cos 2\psi \cos^2 \gamma \, ds$$
$$u = \int \rho \sin 2\psi \cos^2 \gamma \, ds$$
$$i = \int \rho \, ds$$

Uniform distributions of T and dust alignment degree → Polarization angle and polarization degree

Distribution Function of Angle b/w B and Filament axis --- statistical analysis



Planck intermediate results. XXXV (2015).

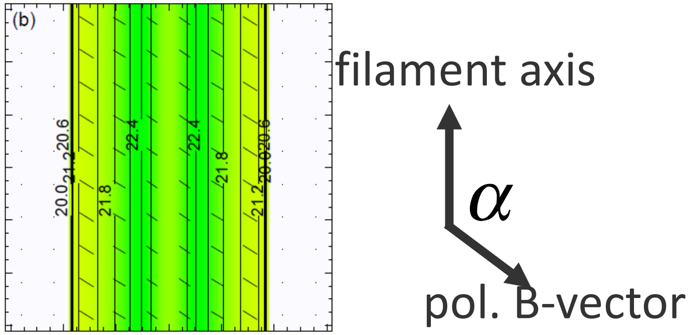
➔ α~90 deg for high-density portion log N>22.08

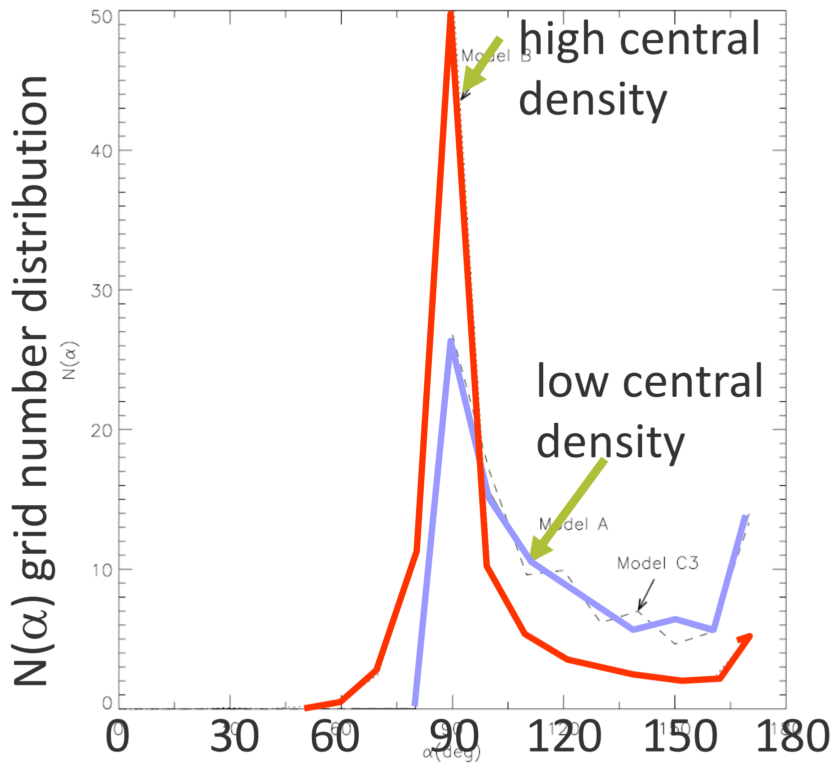
ex)

line of sight

θ=30deg,

ϕ=45deg





α (angle b/w filament axis and pol B-vector)

If all the filaments are observed as perpendicular pol. configuration, filaments may have a high density contrast.

Even when B perp filament in 3D, for some cases, filaments seem to have pol vectors parallel to them.

Dynamical Stability

Linear perturbation problem is hard to be solved, since the Eigenfunction is 2D.

Numerical simulation using AMR code SFUMATO with T. Matsumoto

(A) Random density perturbation is added to each grid point

$$\rho = \rho_{equil}(x_i, y_j) + \delta\rho(x_i, y_j, z_k)$$

δρ/ρ obeys Gauss distribution

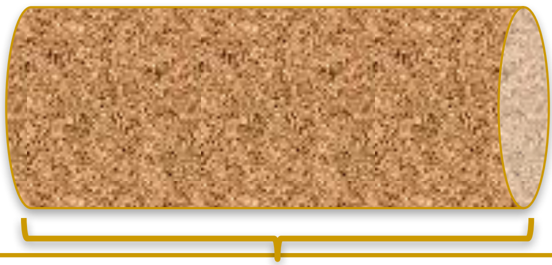
$\overline{\delta\rho / \rho} = 0$ SD =0.1,0.01 made by normal random number

(B) Sinusoidal density perturbation is added

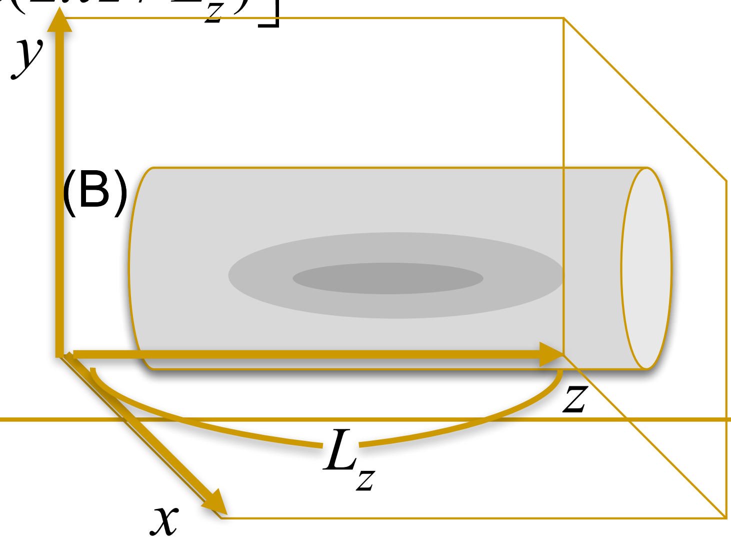
$$\rho = \rho_{equil}(x_i, y_j)\left[1 + A\cos(2\pi z / L_z)\right]$$

$$A = 0.1,\ 0.01$$

(A)

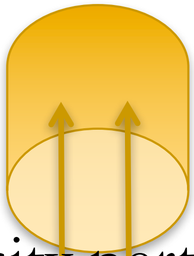


periodic boundary
for the z-direction



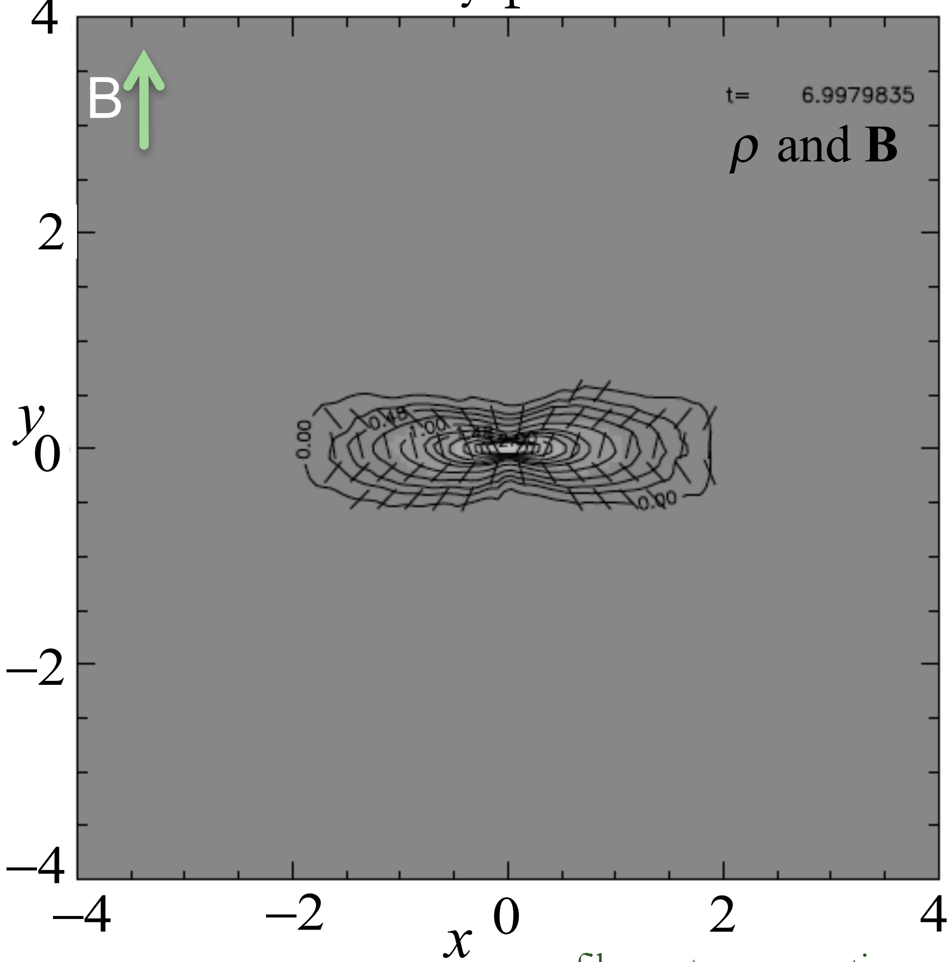
Dynamical Stability

$\beta_0 = 1$
$R_0 = 2$
$\rho_c = 10$

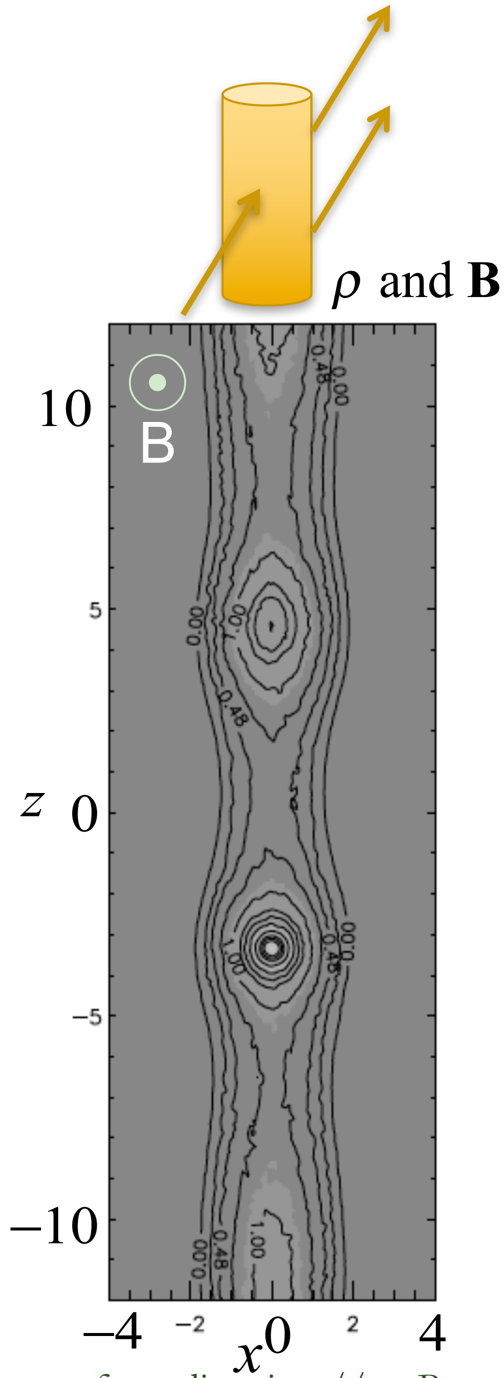


$L_x = L_y = 8$
$L_z = 24$

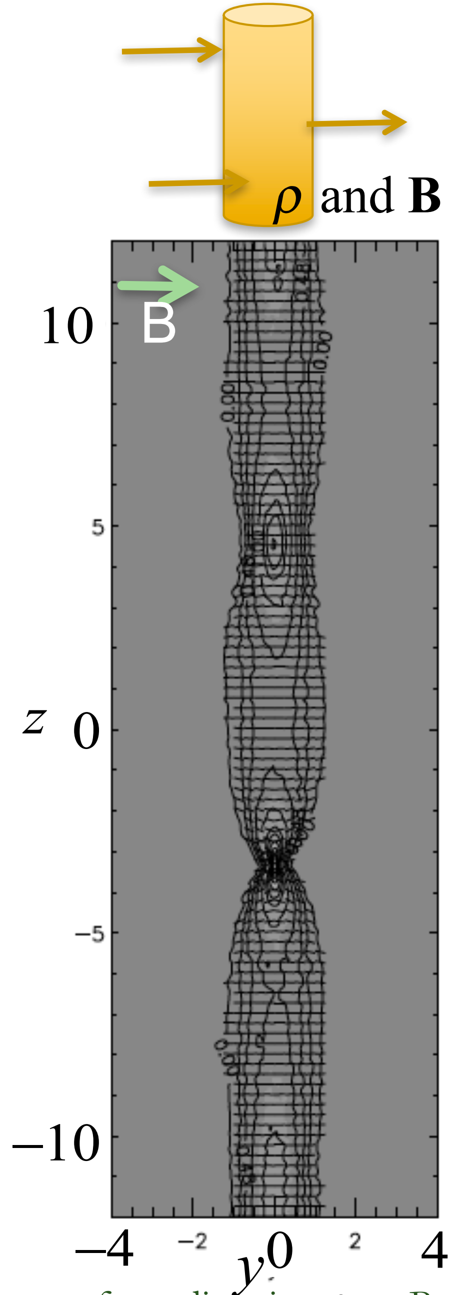
random density perturbation



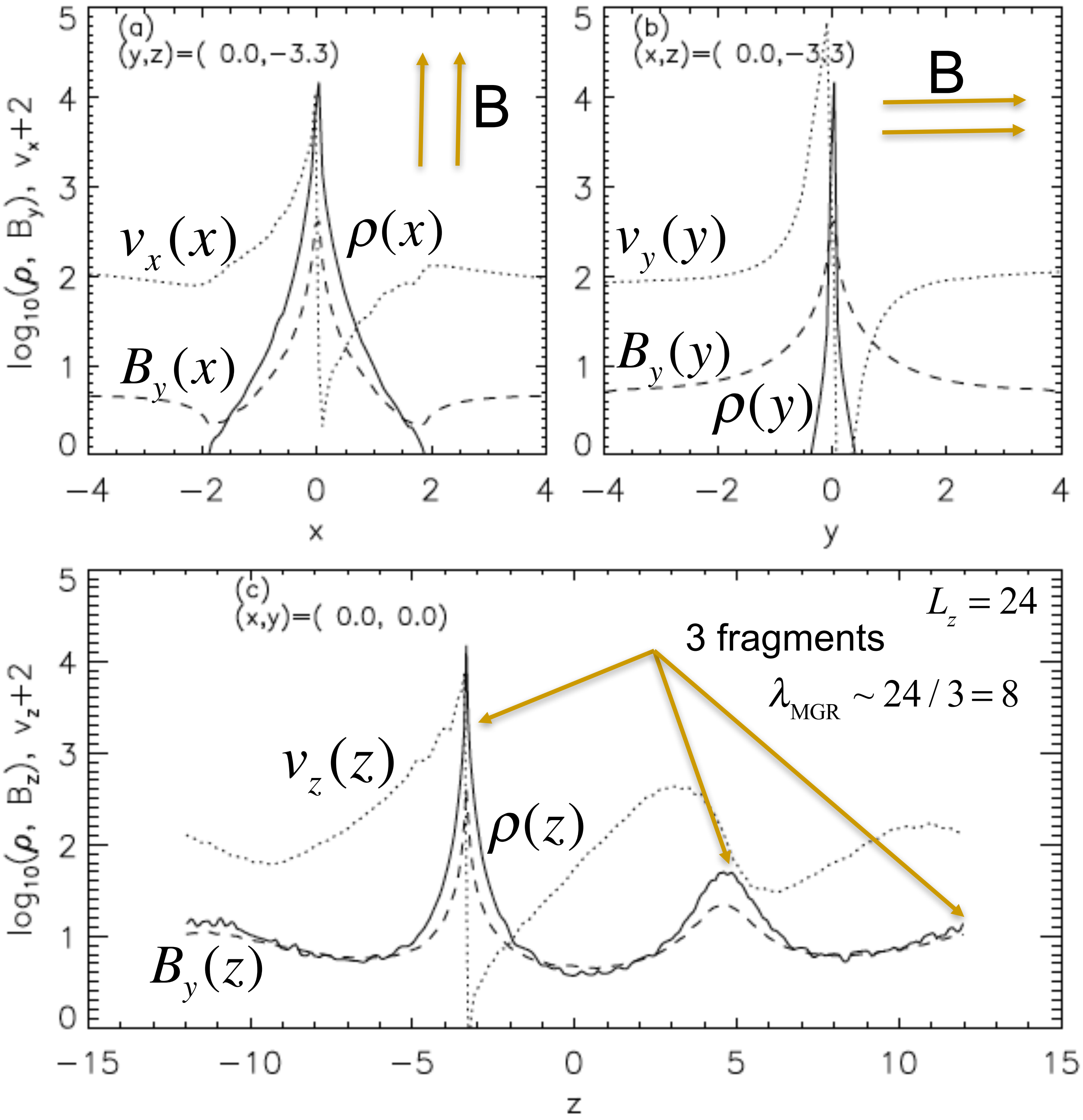
filament cross-section



from direction // to B

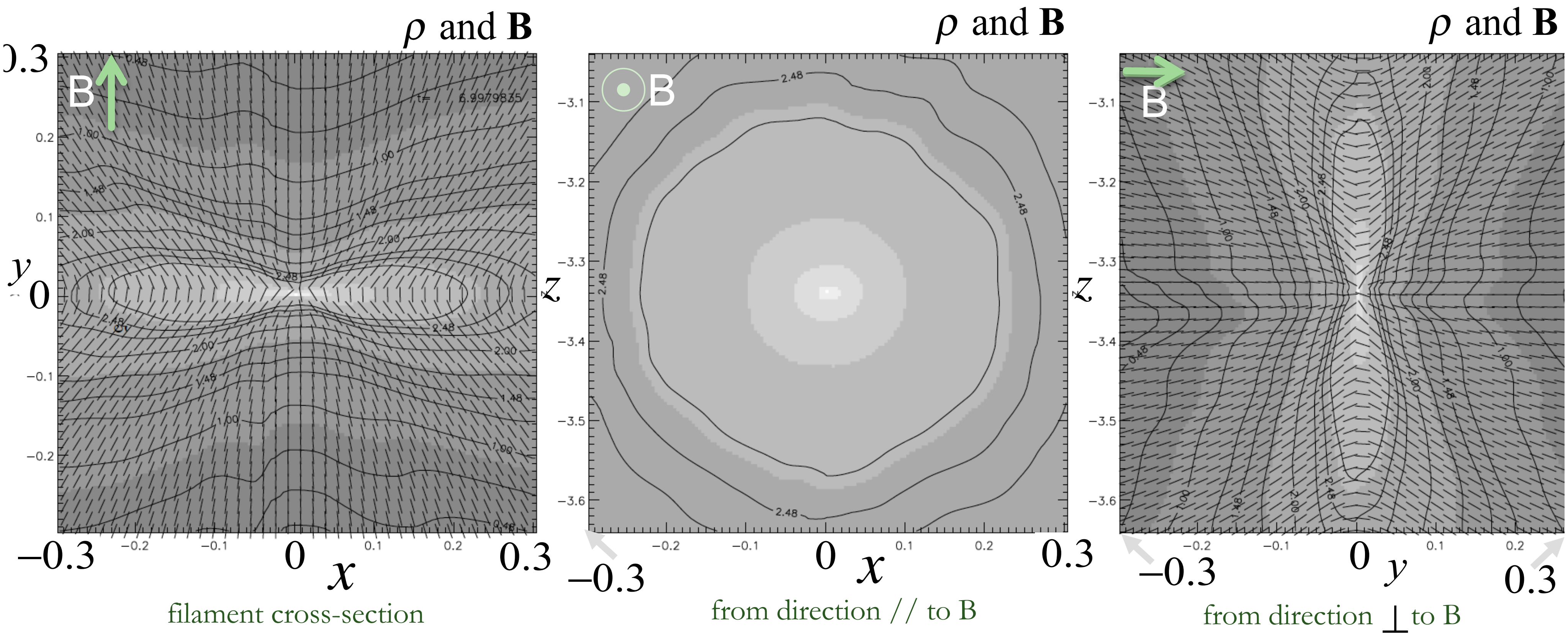


from direction ⊥ to B

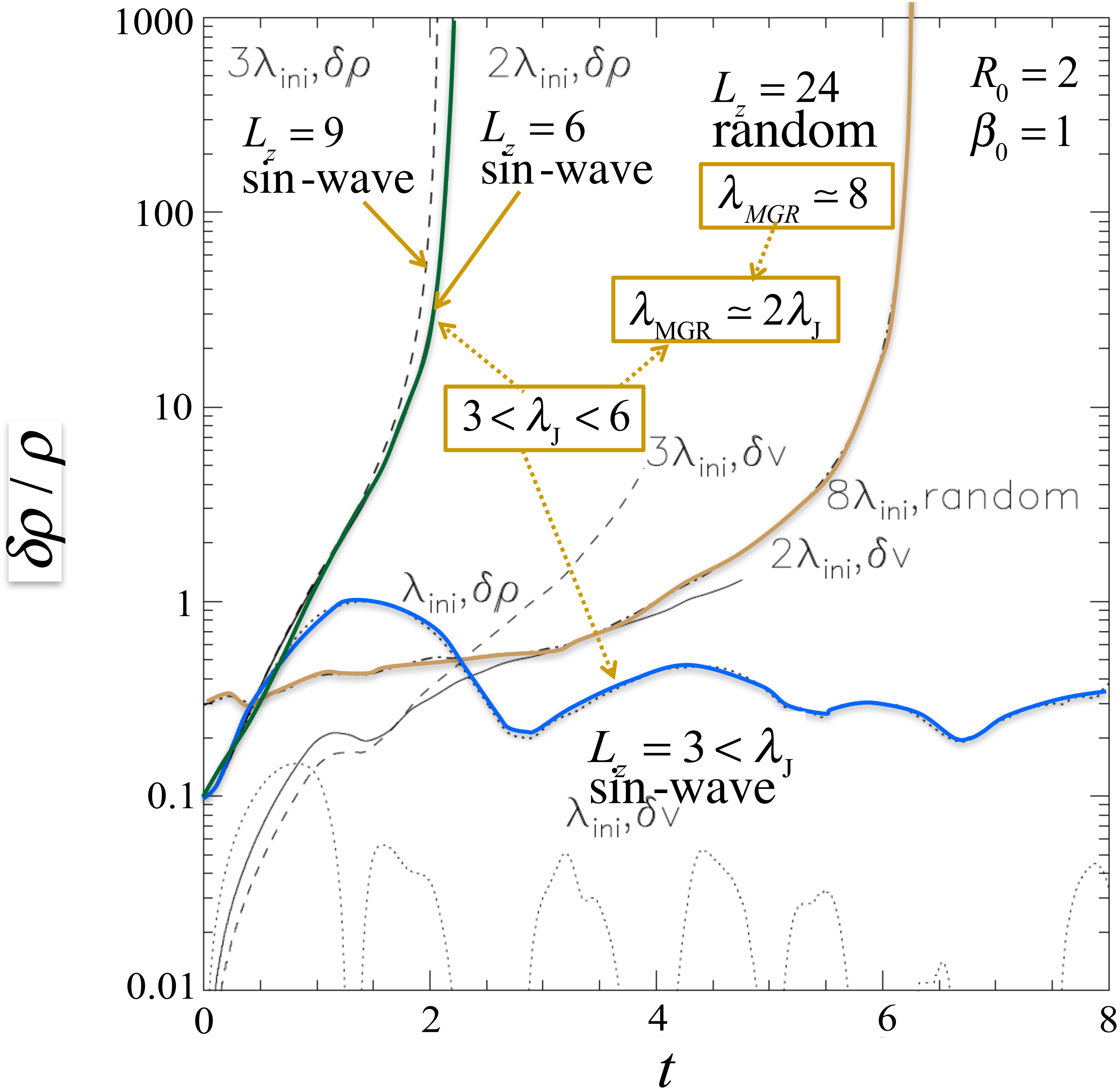


(a)
(y,z)=(0.0,−3.3)
B
$v_x(x)$
$\rho(x)$
$B_y(x)$
$\log_{10}(\rho,\ B_y),\ v_x+2$
x
(b)
(x,z)=(0.0,−3.3)
B
$v_y(y)$
$B_y(y)$
$\rho(y)$
y
(c)
(x,y)=(0.0, 0.0)
$L_z = 24$
3 fragments
$\lambda_{\rm MGR} \sim 24/3 = 8$
$v_z(z)$
$\rho(z)$
$B_y(z)$
$\log_{10}(\rho,\ B_z),\ v_z+2$
z

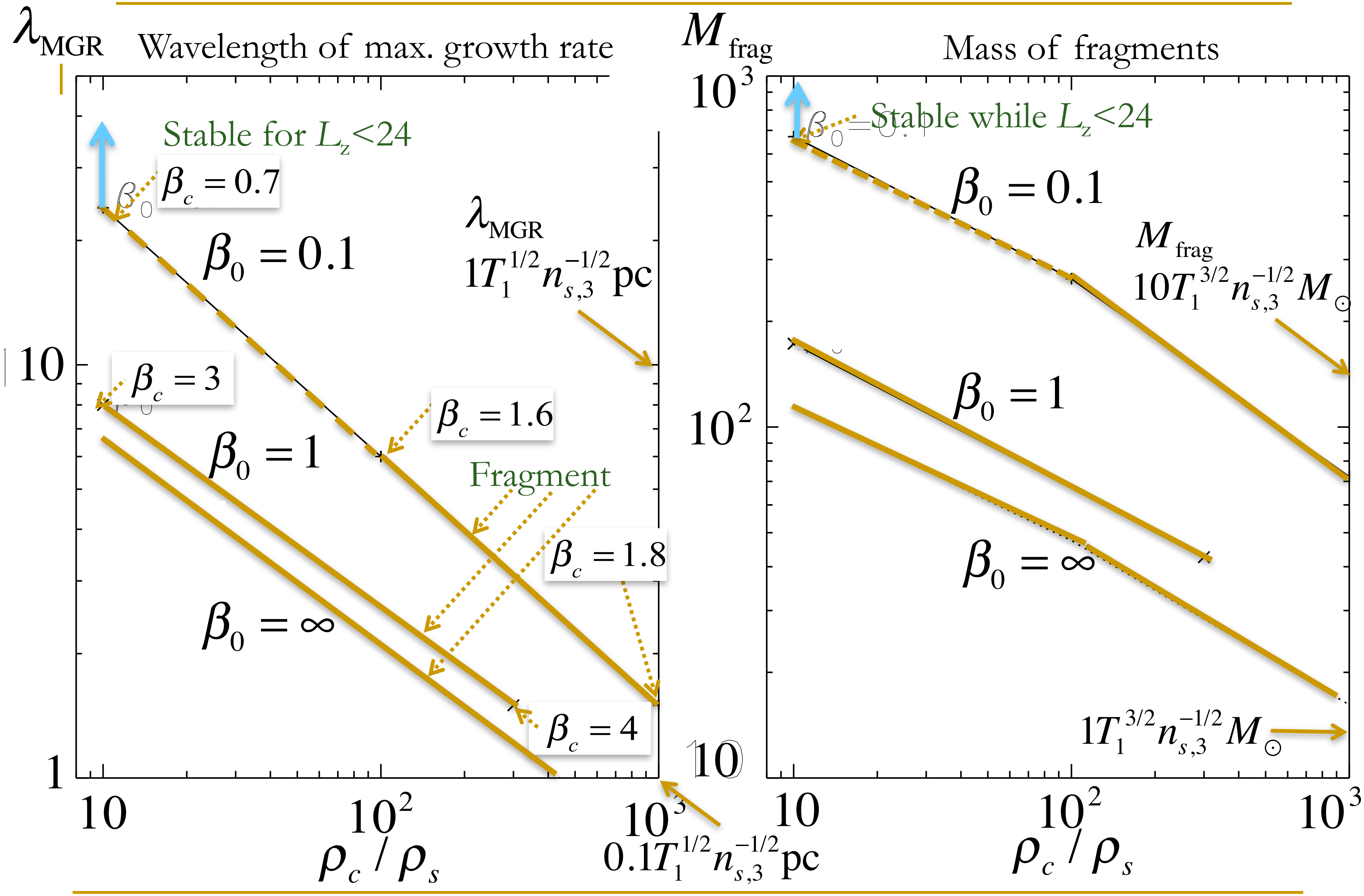
A Pseudo-disk in Runaway Collapse



➔Formation of a Contracting Pseudo-disk



1000
100
10
1
0.1
0.01
0
2
4
6
8
t
δρ / ρ
3λini,δρ
2λini,δρ
$L_z = 9$
sin-wave
$L_z = 6$
sin-wave
$L_z = 24$
random
$R_0 = 2$
$\beta_0 = 1$
$\lambda_{MGR} \simeq 8$
$\lambda_{MGR} \simeq 2\lambda_J$
$3 < \lambda_J < 6$
3λini,δv
8λini,random
2λini,δv
λini,δρ
$L_z = 3 < \lambda_J$
sin-wave
λini,δv



λ_{MGR}
Wavelength of max. growth rate
Stable for L_z<24
$\beta_c = 0.7$
$\beta_0 = 0.1$
λ_{MGR}
$1 T_1^{1/2} n_{s,3}^{-1/2}$ pc
$\beta_c = 3$
$\beta_c = 1.6$
$\beta_0 = 1$
Fragment
$\beta_c = 1.8$
$\beta_0 = \infty$
$\beta_c = 4$
10
1
10^2
10^3
ρ_c / ρ_s
$0.1 T_1^{1/2} n_{s,3}^{-1/2}$ pc
M_{frag}
Mass of fragments
10^3
Stable while L_z<24
$\beta_0 = 0.1$
M_{frag}
$10 T_1^{3/2} n_{s,3}^{-1/2} M_\odot$
10^2
$\beta_0 = 1$
$\beta_0 = \infty$
$1 T_1^{3/2} n_{s,3}^{-1/2} M_\odot$
10
ρ_c / ρ_s

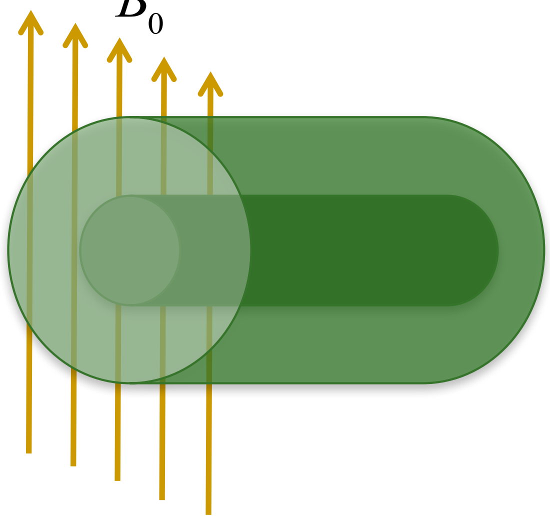
Dispersion Relation of Gravitational Instability
Isothermal cylinder with uniform B-field

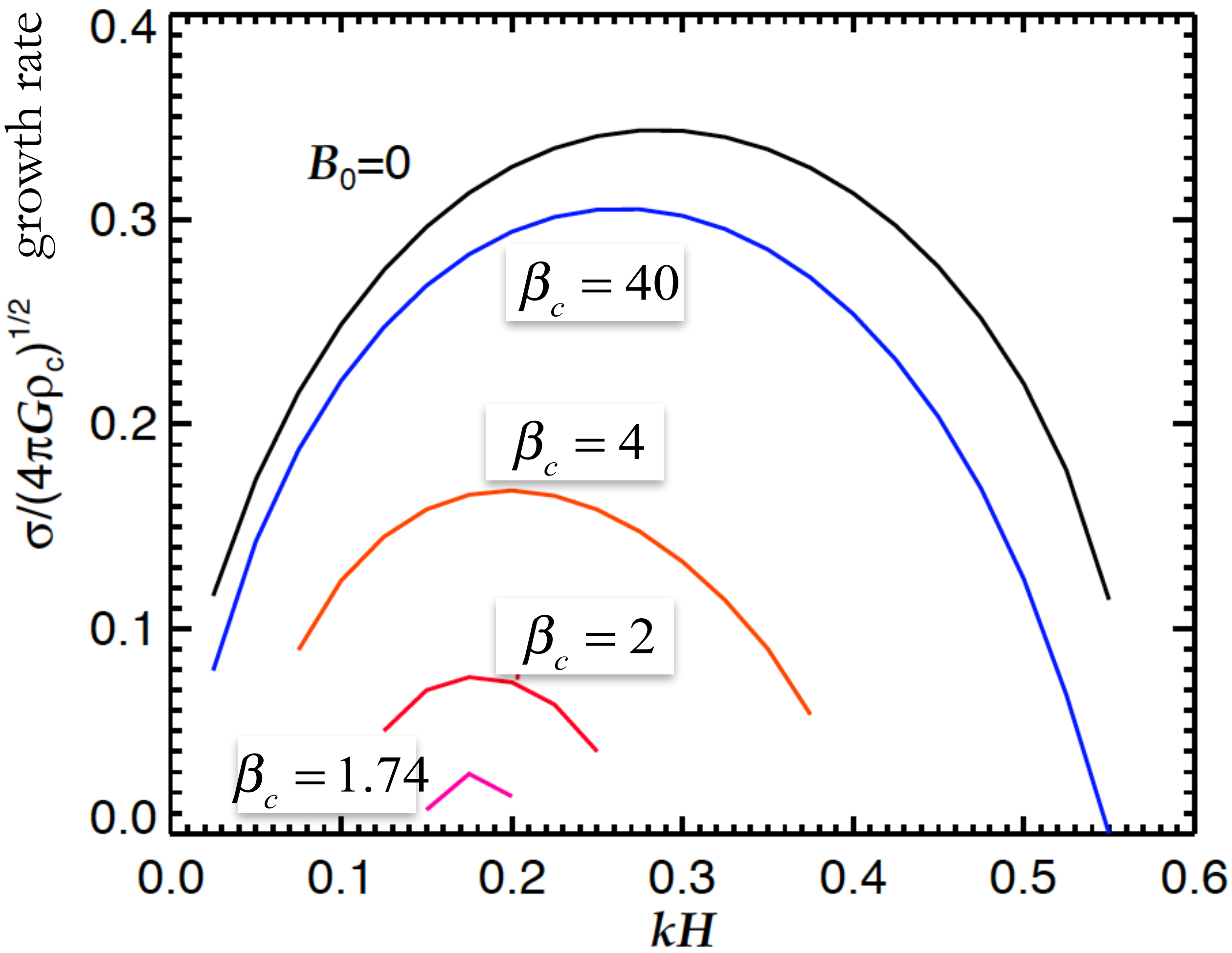
Simplification → Uniform B-field + P_{ext}=0

Eigen-function is 2D

$$\rho(r) = \rho_c \left(1 + \frac{r^2}{8H^2}\right)^{-2}$$

$$\beta_c = \frac{8\pi\rho_c c_s^2}{B_0^2}$$





Condition for stability

$$\beta_c < 1.67$$

Hanawa, Kudoh, Tomisaka (2017) submitted to ApJ

Conclusion

Stability of Filament Threaded by Perpendicular B-Field

for $\lambda_0 < \lambda_{Max}(\Phi_{\mathrm{1D}}, c_s)$

Gravitational Instability

(1) $\lambda < \lambda_{\mathrm{J}}$ stable oscillation

(2) $\lambda > \lambda_{\mathrm{J}}$ gravitational instability

(3) filament fragments into $\lambda_{\mathrm{MGR}} \simeq 2\lambda_{\mathrm{J}}$

Instability may be suppressed for small β_c

Typical scales: Separation ~ $(0.1-1) T_1^{1/2} n_{s,3}^{-1/2}\,\mathrm{pc}$

Mass ~ $(1-10) T_1^{3/2} n_{s,3}^{-1/2} M_{\odot}$

$$\begin{cases} n_{s,3} \equiv n_s / 10^3\,\mathrm{cm}^{-3} \\ T_1 \equiv T / 10\mathrm{K} \end{cases}$$